CWE-GAM METHODOLOGY REPORT GEM-CARE: A GENDERED DYNAMIC GENERAL EQUILIBRIUM MODEL FOR ANALYSIS OF CARE

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CWE-GAM Methodology Report: 21-03
Program on Gender Analysis in Economics (PGAE)
American University, Washington, DC
DOI: 10.17606/3PXK-9060
JUNE 2021

This Methodology Report is part of the Care Work and the Economy (CWE-GAM) Project, based at American University's Program on Gender Analysis in Economics (PGAE). The authors would like to thank CWE-GAM advisors and researchers who provided guidance and feedback throughout the qualitative field work process, as well as Gallup Korea team of organizers, interviewers and transcribers for their assistance with the field work. The authors acknowledge the William and Flora Hewlett Foundation for its Financial support. The findings, views, and conclusions or recommendations expressed in this material are strictly those of the authors and do not necessarily reflect those of the William and Flora Hewlett Foundation.

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THE CARE WORK AND THE ECONOMY (CWE-GAM) PROJECT

The Care Work and the Economy (CWE-GAM) Project strives to reduce gender gaps in economic outcomes and enhance gender equality by illuminating and properly valuing the broader economic and social contributions of caregivers and integrating care in macroeconomic policymaking toolkits. We work to provide policymakers, scholars, researchers and advocacy groups with gender-aware data, empirical evidence, and analytical tools needed to promote creative, gender-sensitive macroeconomic and social policy solutions. In this era of demographic shifts and economic change, innovative policy solutions to chronic public underinvestment in care provisioning and infrastructures and the constraints that care work places on women's life and employment choices are needed more than ever. Sustainable development requires gender-sensitive policy tools that integrate emerging understandings of care work and its connection with labor supply, and economic and welfare outcomes.

Find out more about the project at www.careworkeconomy.org.

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1. INTRODUCTION

The purpose of this paper is to introduce and serve as a reference for GEM-Care, a gendered dynamic CGE model designed for country-level policy analysis with special focus on issues relevant to care in aging East Asian high-income societies like South Korea. The starting point for the model specification is GEM-Core (Cicowiez and Lofgren 2017), which in its turn draws on Lofgren et al. (2013) and Lofgren et al. (2002). GEM-Core has been extended and adapted to the requirements of care and gender analysis, benefitting from the literature on gendered CGE modeling, pioneered by Fontana and Wood (2000) and surveyed in Fontana (2014). Like GEM-Core and its predecessors, GEM-Care may be classified as a multi-purpose template model since, while it includes specific features important to this focus, it can address the wider range of topics that typically are relevant for CGE analysis, including growth, fiscal space, external shocks, poverty, and inequality.[1] The dynamics of the model is of the recursive nature – i.e., actors are assumed to be myopic, making decisions on the basis of data for the current year, which are influenced by past decisions. In addition to dynamic analysis, the model may also be run in a static mode for comparative static analysis. The databases to which GEM-Care is applied may have different aggregations, ranging from relatively aggregate macro applications to much more disaggregated analysis. When applied to issues related to gender and care, the care- and gender-relevant sections of the database have to be disaggregated sufficiently for meaningful analysis of the issues at hand. We proceed as follows: Section 2 of this paper provides a non-technical overview of GEM-Care and its database while Section 3 presents a full mathematical statement.[2]

2. NON-TECHNICAL OVERVIEW OF MODEL STRUCTURE AND DATABASE

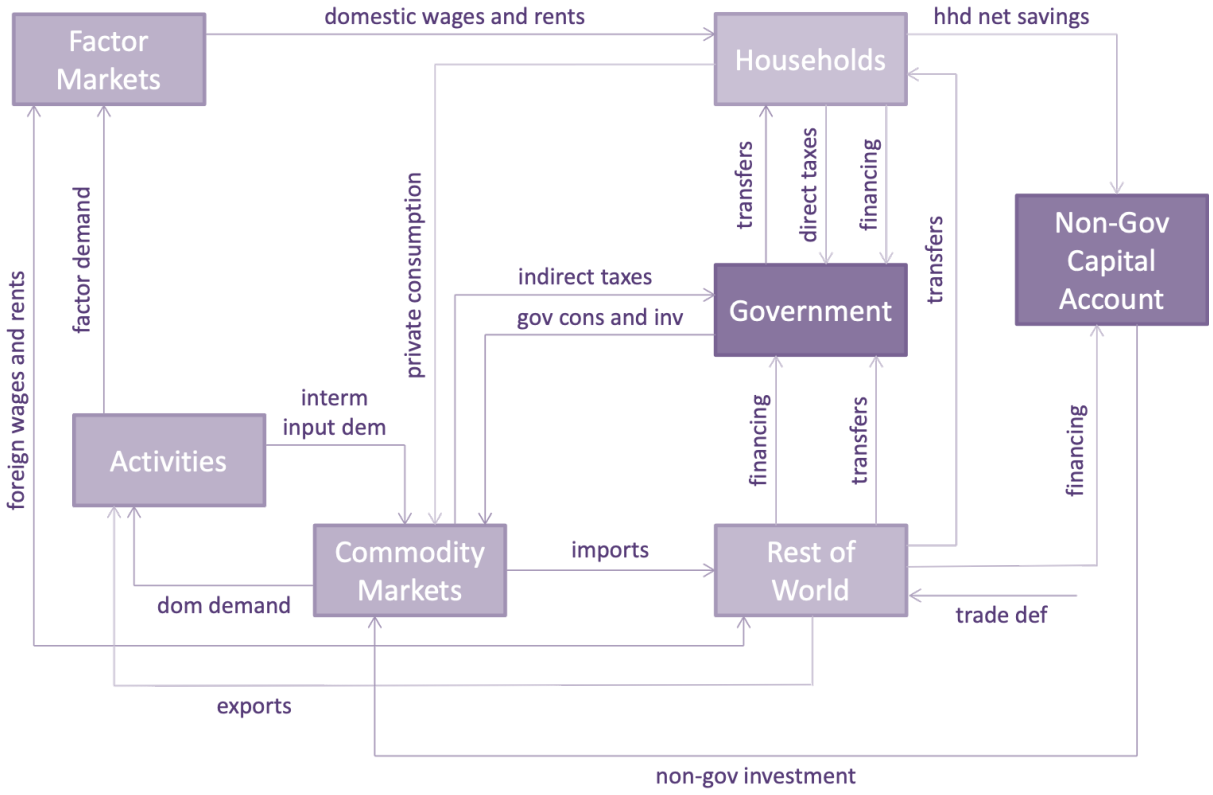
2.1. MODEL STRUCTURE – STATIC MODULE

The static module covers a single year and may also be referred to as the within-period module. This module represents the complete model for the case of comparative static analysis. Figure 2.1 shows the structure of the payments covered by the static module of GEM-Care. Drawing on the Social Accounting Matrix (SAM) format, Table 2.1 displays the same structure with a

[1] Other multi-purpose template models for country analysis include Agénor et al. (2007), Cicowiez and Lofgren (2017), Decaluwé et al. (2013), Lofgren et al. (2002), and McDonald (2015).
[2] GEM-Care is programmed in GAMS (the General Algebraic Modeling System). See www.gams.com.

slightly finer disaggregation of some of the payments.[3] Instead of having monetary values in the different cells, this table describes the content of the different cells that may have values. Its notation is found in Table 2.2.

Figure 2.1. Overview of GEM-Care



[3] A SAM is a matrix representation of the interrelationships in an economy at the level of individual production sectors, factors, and institutions. Like any standard SAM, the one in Table 2.1 is a square matrix with identical accounts in rows and columns. The cells show payments from the column account to the row account. The sum of the entries in a column represents the total expenditures of the column account while the sum of the entries in its row shows the total receipts of the row account. Because of consistency (a feature of the real world and, in the absence of errors in data or concepts, also of any SAM), the row and column totals of each account must be equal. This simply means that no account (or no economic entity) can spend more than it receives, and that any payment received must be used in some way that is captured in the SAM.

Table 2.1. Structure of aggregated SAM

	a-oth-prv	a-oth-gov	a-hser-gdp	a-hser-ngdp	a-leisure-f	a-leisure-m	c-oth-prv	c-oth-gov	c-hser-gdp	c-hser-ngdp	c-leisure-f	c-leisure-m	f-lab-f	f-lab-m	f-cap	tax-indir	tax-dir	hhd	gov	row	cap-hhd	cap-gov	cap-row	inv-prv	inv-gov	dstk	total
a-oth-prv							output																		gfcf	gfcf	dstk	
a-oth-gov								output																			
a-hser-gdp									output																		
a-hser-ngdp										output																	
a-leisure-f											output																
a-leisure-m												output															
c-oth-prv	interm	interm	interm	interm														cons		exp							
c-oth-gov																			cons								
c-hser-gdp																		cons	cons								
c-hser-ngdp																		cons									
c-leisure-f																		cons									
c-leisure-m																		cons									
f-lab-f	va	va	va	va	va															yrow							
f-lab-m	va	va	va	va		va														yrow							
f-cap	va		va																	yrow							
tax-indir	tax		tax				tax		tax																		
tax-dir													cssoc	cssoc	tax			tax									
hhd													yfac	yfact	yfac				trnsfr	trnsfr							
gov															yfac	tax	tax	trnsfr		trnsfr							
row							imp						yfac	yfac	yfac			trnsfr	trnsfr								
cap-hhd																		sav					borr				
cap-gov																			sav		borr		borr				
cap-row																				sav	drf						
inv-prv																					gfcf		fdi				
inv-gov																						gfcf					
dstk																					dstk						
total																											

Table 2.2. Accounts and cell entries in aggregated SAM

Account	Explanation	Cell entry	Explanation
a-oth-prv	activities - non-care private	borr	net borrowing
a-oth-gov	activities - non-care government	cons	consumption
a-hser-gdp	activities - household services GDP	cssoc	social security contributions
a-hser-ngdp	activities - household services non-GDP	drf	change in foreign reserves
a-leisure-f	activities - leisure - female	dstk	change in inventories
a-leisure-m	activities - leisure - male	exp	exports
c-oth-prv	commodities - non-care private	fdi	foreign direct investment
c-oth-gov	commodities - non-care government	gfcf	gross fixed capital formation
c-hser-gdp	commodities - household services GDP	imp	imports
c-hser-ngdp	commodities - household services non-GDP	interm	intermediate inputs
c-leisure-f	commodities - leisure - female	output	domestic production
c-leisure-m	commodities - leisure - male	sav	savings
f-lab-f	labor - female	tax	tax collection
f-lab-m	labor - male	trnsfr	transfers
f-cap	capital	va	value added
tax-indir	tax - indirect	yfac	factor income to institutions
tax-dir	tax - direct	yrow	factor income from RoW
hhd	institutions - households		
gov	institutions - government		
row	institutions - rest of the world		
cap-hhd	capital account - households		
cap-gov	capital account - government		
cap-row	capital account - rest of the world		
inv-prv	investment - private GFCF*		
inv-gov	investment - government GFCF*		
dstk	change in inventories		

(*) For the cell sam('cap-hhd','cap-row'), also including non-FDI foreign investment.

Source: Authors' elaboration.

In Figure 2.1, which serves as the main reference point for this model overview, the arrows show the direction of payments. Except for transfers, these payments are made in exchange for something else that flows in the opposite direction (like the provision of access to a good for consumption in the current period). The major building blocks in the figure are activities (the entities that carry out production), commodities (goods and services produced by activities and/or provided via imports), factors, and institutions (households, enterprises, the [general]

government, and the rest of the world). In applications, most blocks are disaggregated. Similarly, in a SAM that feeds data to GEM-Care, most accounts also tend to be more disaggregated. Given the relatively detailed treatment of the financing of private investment (compared to most other CGE models), the private (non-government) capital account also has its own box.

The following presentation of the model structure assumes that we are dealing with an application to gendered care analysis, i.e. a model that covers both GDP and non-GDP production. To indicate this, we include a-hser-ngdp and c-hser-ngdp among the activities and the commodities in Table 2.1.[4] GDP production is in this case also referred to as production for the market.[5] In this context, it is essential that the activities and the commodities have a sufficient disaggregation of household services, among these singling out care for the young and elderly. Among the factors, labor here refers to all time use that is covered by (and endogenous in) the model, including time spent on leisure and production within and beyond GDP. It does not cover the time needed to meet minimum personal maintenance needs (like sleeping and personal hygiene), which are viewed as non-discretionary and are implicitly exogenous, thus outside the domain of the model. It is likewise essential to disaggregate it by gender – see factors f-lab-f and f-lab-m in Table 2.1. However, it may be noted that this representation of the economy is very general in the sense that, apart from the specifics of account disaggregation and the pattern of non-zero cells, it also captures what may be done in a macro application with a small number of accounts where production is limited to what is part of GDP, or a highly disaggregated application that also covers part of non-GDP production.

Starting with the activities, as indicated by Figure 2.1, they produce and sell their output in (domestic) commodity markets or to the rest of the world (as exports). The activities use their revenues to cover costs of intermediate inputs and to pay wages and rents to factors.
The only factor used by government activities is labor, while private activities tend to use both labor and private capital. For government activities, the output level tends to be determined by government demand, a policy tool, which in its turn determines labor hiring and intermediate input demand. For the private sector, profit maximization drives decisions regarding factor employment, which determine the output level and intermediate demands.[6] The split of private

[4] Under the System of National Accounts, production that is part of GDP is referred to as being "within the production boundary". It includes (a) all production actually destined for the market or provided for free by the government or NPISHs (non-profit institutions in the service of households); (b) household production of *goods* that are retained for final consumption within the household (such as production of agricultural goods, or of a house); and (c) the production of *housing services* for own final consumption by owner occupiers. It does not include the production of domestic and personal services for consumption within the same household (such as preparation of meals and care and training of children) (UN 2009:6-7).
[5] This is not fully accurate since GDP production also includes the services of owner-occupied housing and goods produced by households for own consumption (cf. preceding footnote).
[6] In terms of production technology, for both sectors, at the top level of the production nest, intermediate input demand and aggregate factor demand are fixed coefficients per unit of output. At the lower level, the

output between exports and domestic sales depends on relative sales prices in these two destinations.

Among the institutions, the households (which may be disaggregated) earn incomes from factors, (net) transfers from the government, and (net) transfers from the rest of the world.[7] After paying direct taxes (determined by policy), the household spends in fixed shares on private commodity consumption (which is defined broadly to include not only GDP commodities but also non-GDP commodities and leisure) and savings. The allocation of consumption spending across different commodities is specified by demand functions derived from utility maximization. After deducting net financing of the government and of changes in foreign reserves, household savings are used to finance private investment.

Within this structure, household services (like care and other services like cleaning and cooking) are of particular importance to the current application. Household services are services that are produced by a production activity that uses household labor and supplies its output for use by the household that provided the labor. More specifically, these services are viewed as being sold by the household activities in the (domestic) commodity market for private consumption by the labor-providing household. Household consumption demand is nested so that, for services with both household and market sources of supply (i.e., c-hser-gdp and c-hser-ngdp, respectively in Table 2.1), the split between demands for services across the two supply sources depends on relative prices. If the only input in the production of a household service is labor (a likely treatment in the absence of data on the use of other inputs), the imputed sales revenue is identical to the imputed income earned by household labor.[8] The unit price of the service is determined by the cost of the inputs used (mainly labor). The fact that household production and consumption of services are treated as part of a more general structure has the virtue of making it possible to enrich the model considerably with only a minor cost in complexity. It also makes it possible to draw on these extensions in other but structurally similar contexts of consumption choice, production input choice, and discrimination. (This is discussed further in Section 3).

The government (as an institution, not its service production activities) gets its receipts from taxes, transfers from abroad, and net financing from households and the rest of the world. It uses these receipts for transfers to households, consumption, and investment (to provide the capital stocks required for government services). To remain within its budget constraint, it either adjusts some part(s) of its spending on the basis of available receipts or mobilizes additional receipts of one or more types to finance its spending plans.

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substitutability between labor and capital for the private sector is determined by a CES (Constant Elasticity of Substitution) function. See Section 3 for more detail on functional forms.

[7] In Figure 2.1, transfers are implicitly netted (since they only go in one direction) and may therefore be negative. In the model and its database, it is possible to include transfers in both directions.

[8] The latter statement is not true if the household service uses intermediate inputs; if so, the labor income falls short of the sales revenue.

The rest of the world receives and makes the payments that appear in the balance of payments. As shown in Figure 2.1, imports are represented by payments from commodity markets to the rest of the world while exports appear in the form of payments from the rest of the world to activities. According to the stylized SAM in Table 2.1, all government output is sold at home while private (non-government) output is both sold at home and exported; this is also largely true in most applied analysis. However, among private commodities, some may be non-traded or only traded in one direction. Foreign wages and rents are the only non-trade payments to the rest of the world. The non-trade payments received from the rest of the world are net transfers and financing to government and the private sector; the latter also includes foreign investment other than FDI. In a dynamic application, non-trade payments are typically based on exogenous projections.

For the government commodity (produced by the government activity), the price paid for the demand-driven supply quantity depends on the unit supply cost (for labor and intermediate inputs). In the markets for private commodities, flexible prices ensure balance between demands for domestic output from domestic demanders and supplies to the domestic market from domestic suppliers. Part of domestic demands for the private commodity are for imports; the ratio between demands for imports and domestic output depends on the ratio between the demander prices for commodities from these two sources (i.e., the prices demanders pay including relevant taxes and trade and transport margins) – an increase in the import/domestic price ratio lowers the ratio between the demands for imports and domestic output (and vice versa). Similarly, domestic suppliers (the activities) also consider relative prices when deciding on the allocation of their output between the domestic market and exports. For both exports and imports, the standard assumption is that international prices are exogenous (the small country assumption).[9]

These import and export responses to relative price changes underpin the standard clearing mechanism for the balance of payments: changes in the real exchange rate (the ratio between international and domestic price levels, which may change due to changes in the nominal exchange rate) influence export and import quantities and values. For example, other things being equal, an exchange rate depreciation may eliminate a balance of payments deficit by raising the export quantity and reducing the import quantity (and vice versa for an appreciation).
Within this structure, household services (like child care provided by female family members) are part of private commodity production for the domestic market. Like other private commodities, their prices are flexible, balancing quantities supplied and demanded. To exemplify, other things being equal, the price of household care would increase if female wages outside the home improve (leading to a leftward shift in the supply curve for the service due to a cost increase) and/or if there is an increase in the price of market substitutes to family-provided care (leading

[9] Both for imports and exports, the model offers the option of endogenizing prices (in foreign currency) using constant-elasticity demand and supply functions, respectively.

to a rightward shift in the demand curve). (The above-mentioned nesting of household consumption demand assures that these responses are present; this is explained in detail in Section 3.)

Turning to factor markets, for both labor and capital, the demand curves are downward sloping, reflecting the production activity responses to changes in wages and rents while, within any given time period, the supply is fixed, represented by a vertical supply curve. For labor, this means that there is no explicit reference to unemployment. This follows from the fact that labor here refers to an exogenous quantity of time the allocation of which is endogenous within the model (24 hours per day net of an exogenous allocation of time needed to satisfy basic needs like sleeping, eating, and personal hygiene). Time that in other context would have been spent in unemployment (time supplied to GDP work but not employed) is here explicitly allocated to other uses (leisure or work in non-GDP activities).[10]

2.2. MODEL STRUCTURE – DYNAMIC MODULE

The dynamic (or between-period) module is put into play when the analysis is extended to cover multiple periods. In this setting, the economy grows over time due to employment growth for private capital and labor as well as growth in total factor productivity (TFP). For capital (only used by the private activities), employment growth coincides with stock growth, which depends on investment and depreciation; for labor, employment growth depends on growth in the stock, which in its turn is a function of population growth disaggregated by gender and age. Apart from an exogenous component, the TFP of both the private and the government activity may depend on growth in the public capital stock.

An additional dynamic aspect of the model is related to debt stocks. As noted above, the model covers the following (net) financing flows: to the government from domestic non-government institutions (households and enterprises) and the rest of the world; and to domestic non-government institutions from the rest of the world. On the basis of the results for any simulation, assumptions about real interest rates, and initial debt stocks, post-calculations extract the implications of financing for the evolution of domestic and foreign debt (or asset) stocks. The same applies to the evolution of the stock of foreign reserves, which is computed on the basis of the initial stock and annual changes.

[10] While the aggregate labor (or time) supply is vertical, the supply curve for GDP labor is upward sloping – other things being equal, a higher wage in GDP activities leads to a reallocation of time to these activities.

2.3. AREAS OF POLICY ANALYSIS

A model like GEM-Care can help analysts better understand the effects of a wide range of policies and exogenous shocks. Beyond the concern of the current project with care policies in an East Asian high-income setting, it may for example, analyze the space for government consumption and investment spending under alternative scenarios for TFP growth and taxation, considering budgetary and sustainability constraints. Depending on the specifics of the application, it may also capture the effects of different types of government spending that are simulated. Alternatively, it could consider the impact of reliance on alternative funding source for a planned spending program. Outside the fiscal area, the model may also be used to assess the consequences of shocks affecting world (export and/or import) prices, migration flows, and worker remittances (current private transfers). It is straightforward to address demographic issues, including the impact on growth and living standards of changes in population size and age structure (endogenous or migration driven). It is also straightforward to combine shocks in different areas – for example, how do alternative scenarios for export impinge on the ability of a government to fund different programs.

While this may be less relevant for an application to South Korea, GEM-Care can also assess the poverty and distributional effects of different scenarios, drawing on a built-in poverty module based on Lofgren et al. (2013). The module offers a choice between the following approaches: (i) constant elasticity of poverty with respect to per-capita welfare for each model household; (ii) log-normal distribution of per-capita welfare within each model household; and (iii) distribution of per-capita welfare within each model household following a real-world household survey. The module is linked to base-year poverty and distributional data for each of the representative households (RHs; one or more) in the database. It uses either household income or consumption as its welfare measure. The ability of the module to account for distributional change and its impact on poverty depends on the degree of disaggregation of the RHs. In applications with a single RH, it merely projects poverty outcomes on the assumption that distribution does not change.[11]

2.4. DATABASE

The disaggregation of any application is determined by its database, which is most conveniently contained in a single Excel file. The major components of the data for a static application of GEM-Care to gendered care analysis are a social accounting matrix (SAM), physical data on gendered time use (labor and leisure), population data, and a set of elasticities (related to production, trade, and household consumption). The role of the SAM is to define base-year values for the bulk of

[11] GEM-Care may also feed data to a separate microsimulation module to generate poverty and distributional indicators.

the model parameters, including those covering production technologies, sources of commodity supplies (domestic output or imports), commodity demands (for household and government consumption, investment, stock change, and exports), transfers between different institutions, and tax rates. For a dynamic application, the base run (to which other scenarios are compared) requires projections for the population (by age and gender), GDP growth, and government policies. If the base run is viewed as a "business-as-usual" scenario, then the growth in different spending and revenues items may simply be set to growth at the same pace as the overall economy. GDP growth projections may be based on the trend from the recent past.

The types of analysis to which the model can be applied depend on the disaggregation of the SAM and the rest of the database. In order to analyze care-related issues, it is crucial to have a disaggregated representation of care-related production and time use. Thanks to the fact that GEM-Care is a general equilibrium model, care will then be considered in the context of the rest of the economy, including the decisions and budget constraints of households, producers, and the government. For households, this broader context also includes alternative time uses for its members in the context of a time constraint.

3. GEM-CARE MATHEMATICAL STATEMENT

This section presents a mathematical statement of GEM-Care, showing the relationships that, together with the database, determine the results of model simulations. A good understanding of the structure of the model and its database is needed to understand the simulation results. The presentation is organized around a set of tables and is divided into two subsections: notation (3.1) and equations (3.2).

3.1. NOTATION

Table 3.1.1 explains notational principles, designed to facilitate understanding the statement. Tables 3.1.2-3.1.5 define model sets, variables, Latin-letter parameters, and Greek-letter parameters, respectively. In each of these tables, the items are arranged alphabetically. Given that this model is dynamic, a time index is part of the domains of all variables and the parameters that are most likely to change over time.
All model components are potentially active but whether they are used in any given application depends on the disaggregation of the database.

Table 3.1.1. Notational principles

Items	Notation	Example
Sets	Lower-case Latin letters as subscripts to variables and parameters	See the following rows
Endogenous variables	Upper-case Latin letters (without a bar)*	$QG_{c,t}$
Exogenous variables**	Upper-case Latin letters with a bar*	$\overline{QFS}_{f,t}$
Parameters**	Lower-case Latin letters* or lower-case Greek letters (with or without superscripts)	$ica_{c,a}$;ρ_c^q

*The names of Latin letter variables and parameters that refer to prices, quantities, and factor wages (rents) start with P, Q, and WF, respectively.
**The distinction between exogenous variables and parameters is that the latter always have exogenous values whereas the former under alternative assumptions may be endogenous.

Table 3.1.2. Sets

Name	Description
$a \in A$	activities (production sectors or industries)
$a \in AGDP(\subset A)$	GDP activities
$a \in ANGDP(\subset A)$	non-GDP activities (leisure and household service activities)
$c \in C$	commodities (i.e., goods and services)
$c \in CD(\subset C)$	commodities with domestic sales of domestic output
$c \in CE(\subset C)$	exported commodities
$c \in CM(\subset C)$	imported commodities
$c \in CT(\subset C)$	transactions commodities (services paid under distribution margins)
$c \in C1(\subset C)$	commodities at level 1 of utility function
$c \in C2(\subset C)$	commodities at level 2 of utility function
$c \in CSAM \subset C$	commodities in SAM
$c \in CNSAM \subset C$	commodities not in SAM
$f \in F$	factors

$f \in FVA (\subset F)$	factors that earn value added (in SAM)
$f \in FCAP (\subset F)$	capital factors
$f \in FCAPG (\subset FCAP, \notin FVA)$	gov't capital factors (do not earn value-added)
$f \in FCAPNG (\subset FCAP, \subset FVA)$	non-gov't capital factors (earn value-added)
$f \in FLAB (\subset FVA)$	labor factors (earn value-added)
$f \in FNSAM (\subset F)$	factors not in SAM
$f \in FOTH (\subset FVA, \notin FLAB, \notin FCAP)$	other factors (earn value-added; not capital or labor)
$f \in FSAM (\subset F)$	factors in SAM
$f \in F1 (\subset F)$	factors at level 1 (top) of nest
$f \in F2 (\subset F)$	factors at level 2 of nest
$i \in INS$	institutions
$i \in INSD (\subset INS)$	domestic institutions
$i \in INSDNG (\subset INSD)$	domestic non-government institutions
$i \in INSNG (\subset INS)$	non-gov't institutions (rest of world and elements in INSDNG)
$h \in H (\subset INSDNG)$	households
$(c, c') \in MC2C1$	mapping between c in C2 and c' in C1
$(f, f') \in MF2F1$	mapping between f in F2 and f' in F1
$(h, angdp) \in MHANGDP$	mapping between households and non-GDP activities
$t \in T$	time periods (simulation years)
$t \in TMIN$	base period (first simulation year)
$tac \in TAC$	transactions (distribution) types (domestic, import, export)
$tacd \in TACD (\subset TAC)$	transactions (distribution) for domestic sales
$tace \in TACE (\subset TAC)$	transactions (distribution) for exports
$tace \in TACM (\subset TAC)$	transactions (distribution) for imports

Table 3.1.3. Variables

Name	Description
CPI_t	consumer price index
$DKA_{f,a,t}$	change in capital stock f allocated to activity a
$DKINS_{i,f,t}$	investment by institution i (in INS) in capital stock f
DPI_t	domestic producer price index (PDS-based)
EG_t	total current government expenditure
$EH_{h,t}$	consumption expenditure for household h

EXR_t	exchange rate (local currency per unit of foreign currency
$INV_{i,t}$	value of investment (including stock change) for institution i (in INSNG)
$INVG_t$	value of investment (including stock change) for government
$MPS_{i,t}$	marginal propensity to save for domestic non-government institution i (in INSDNG)
$MPSSCAL_t$	MPS scaling factor
$NFFG_t$	net foreign financing of government (FCU)
$NFF_{i,t}$	net foreign financing for non-government institution i (in INSDNG) (FCU)
$PA_{a,t}$	output price for activity a
$PDD_{c,t}$	demand price for commodity c (in C) produced and sold domestically
$PDS_{c,t}$	supply price for commodity c (in C) produced and sold domestically
$PE_{c,t}$	price for export of c (in C) (LCU) (net of export taxes and distribution margin)
$PK_{f,t}$	price (per unit of) of capital stock f
$PM_{c,t}$	price for import of c (in C) (LCU) (includes import tariffs and distribution margin)
$PQD_{c,d,t}$	composite commodity price for c (in C) for domestic demander (type) d (in D) [includes commodity subsidies, all taxes (including VAT and sales tax), and distribution margins]
$PQS_{c,,t}$	composite commodity price for c (includes import tariffs and distribution margins but not sales tax, commodity subsidies, or VAT)
$PX_{c,t}$	producer price for commodity c
$PVA_{a,t}$	value-added price for activity a
$QA_{a,t}$	level of activity a
$QD_{c,t}$	quantity sold domestically of domestic output c
$QE_{c,t}$	quantity of exports of commodity c (in C)
$QF_{f,a,t}$	quantity demanded of factor f by activity a
$QFINS_{i,f,t}$	endowment of institution i (in INSD) of factor f
$QFS_{f,t}$	supply of factor f to GDP activities
$QFSGDP_{i,f,t}$	supply of factor f from institution i to GDP activities
$QFSNGDP_{i,f,t}$	supply of factor f from institution i (in H) to non-GDP activities
$QG_{c,t}$	quantity of government consumption of commodity c
$QGSCAL_t$	government consumption scaling factor
$QH_{c,h,t}$	quantity consumed of commodity c by household h

$QINT_{c,a,t}$	quantity of commodity c as intermediate input to activity a
$QINV_{c,t}$	quantity of investment demand for commodity c (investment by source)
$QINVSCAL_t$	investment scaling factor
$QM_{c,t}$	quantity of imports of commodity c (in C)
$QQ_{c,t}$	quantity of composite demand (and supply) of commodity c (in C)
$QT_{c,t}$	quantity of trade and transport services demand for commodity c (in C)
$QX_{c,t}$	quantity of domestic output of commodity c (in C)
$RGDPMP_t$	real GDP at market prices (at constant base-year prices)
$SAVF_t$	foreign savings (FCU)
$SAVG_t$	government savings
$SAV_{i,t}$	savings of domestic non-government institution i (in INSDNG)
$SHIF_{i,f,t}$	share for institution i (in INSD) in the income of factor f from GDP activities
$SUBCT_t$	government spending on commodity subsidies
$TFA_{f,a,t}$	rate of tax on use of factor f by activity a
$TFASCAL_t$	scaling of rate of tax on use of factor f by activity a
$TFP_{a,t}$	total factor productivity for activity a
$TFPSCAL_t$	scaling of total factor productivity
$TRDGDP_t$	real foreign trade (exports+imports) and GDP ratio
$TRII_{i,i',t}$	transfers to institution i (in INS) from domestic non-government institution i' (in INSDNG)
$WALRAS_t$	variable check on Walras' law (which is satisfied if value is zero)
$WF_{f,t}$	economywide wage of factor f
$WFA_{f,a,t}$	wage for factor f in activity a
$WFAVG_{f,t}$	average wage for factor f (in FCAPNG)
$WFDIST_{f,a,t}$	wage distortion factor for factor f in activity a
$YFGDP_{f,t}$	GDP income of factor f (from GDP activities and transfers)
$YFNGDP_{i,f,t}$	income of institution i (in H) from factor f in non-GDP activities
YG_t	government current revenue
$YI_{i,t}$	income of (domestic non-government) institution i (in INSDNG)
$YIF_{i,f,t}$	income of institution i (in INSD) from factor f
$YTXDIRI_{i,t}$	direct tax payments by institution i (in INS)

Table 3.1.4. Latin letter parameters

Name	Description
$capcomp_{c,f}$	quantity of commodity c per unit of new capital stock f

$cwts_{c,h}$	weight of commodity c in consumption basked of household h
$depr_{f,t}$	rate of depreciation for capital stock f
drf_{t}	change in foreign reserves (FCU)
$dsc_{f,a,t}$	rate of discrimination against (labor) factor f in activity a
$dwts_{c}$	weight of commodity c in the DPI (PDS-based producer price index)
$ica_{c,a}$	quantity of intermediate input c per unit of activity a
$icd_{c,c'}$	input of c for trade and transportation per unit of commodity c' produced and sold domestically
$ice_{c,c'}$	transactions input of c per unit of commodity c' export
$icm_{c,c'}$	transactions input of c per unit of commodity c' imports
$invshr_{f,i,t}$	share for capital stock f in investment spending of institution i (in INSNG)
$mpsb_{i,t}$	baseline marginal propensity to save for domestic non-gov't institution i (in INSDNG)
$ndfg_{t}$	net domestic financing to government (indexed to numéraire) (FCU)
$nff_{i,t}$	net foreign financing to institution i (in INSD) (FCU)
$pop_{ac,t}$	population of ac (household h in H or country total)
$pwe_{c,t}$	export price for commodity c (in foreign currency)
$pwm_{c,t}$	import price for commodity c (in foreign currency)
$qdstk_{c,i,t}$	change in stock (inventories) of c for institution i (in INSD)
$qfinsb_{i,f,t}$	endowment for institution i (in INSD) of factor f (in FOTH)
$qgb_{c,t}$	baseline quantity of government consumption of commodity c
$qg01_{c,t}$	0-1 parameter turning on-off potential scaling of gov consumption of c
$qinvb_{c}$	base-year quantity of investment (GFCF) demand for c
$shii_{i,i'}$	share of institution i (in INS) in the income (net of direct taxes and savings) of domestic non-gov't institution i' (in INSDNG)
$sub_{c,d,t}$	rate of subsidy on commodity c (in C) for demander d (in D)
$ta_{a,t}$	rate of tax on gross output value for activity a
$te_{c,t}$	rate of tax on commodity c
$tf_{f,t}$	rate of direct tax on factor f
$tfab_{f,a,t}$	base-year rate of tax on use of factor f by activity a
$tfpb_{a,t}$	exogenous component of TFP for activity a
$tm_{c,t}$	rate of import tariff on commodity c
$tq_{c,t}$	rate of sales tax on commodity c
$trnsfr_{ac,i,t}$	transfers from institution i (gov't or rest of world) to ac [where ac is institution i (in INS) or factor f (in F)] (LCU if from gov't; FCU if from rest of world)
$tva_{c,d,t}$	rate of value-added tax on commodity c (in C) for demander d (in D)

$ty_{i,t}$	rate of direct tax on domestic non-gov't institution i (in INSDNG)
wfb_f	exogenous economywide wage term for activity-specific factors
$wfdistb_{f,a}$	exogenous activity-specific wage term for mobile factors

Table 3.1.5. Greek letter parameters

Name	Description
$\alpha_{i,t}^{sav}$	intercept in savings function for institution i (in INSDNG)
$\beta_{c,h}$	share parameter in LES function for household consumption of commodity c
$\gamma_{c,h}^{min}$	minimum quantity in LES function for household consumption of commodity c
δ_c^{dd}	share parameter for domestic purchases in Armington function for commodity c (top of nest)
δ_c^{ds}	share parameter for domestic sales in CET function for commodity c (top of nest)
δ_c^{e}	share parameter for exports in CET function for aggregated commodity c (in C) (top of nest)
δ_c^{m}	share parameter for imports in Armington function for commodity c (top of nest)
$\delta_{c,h}^{qh}$	share parameter for consumption of composite commodity c by household h
$\delta_{f,a}^{va}$	share parameter for factor f in CES VA function for activity a
$\delta_{f,a}^{2}$	share parameter for level 2 of production function for factor f in activity a
$\eta_{a,f}^{tfp}$	elasticity of TFP in activity a with respect to gov't capital stock f
$\theta_{a,c}$	yield of output c per unit of activity a
κ_f	sensitivity of the allocation of new capital for f (in FCAPNG) across activities (in A) to current deviations of activity capital rents from the economywide average
ρ_c^{q}	exponent in Armington function for commodity c
$\rho_{c,h}^{qh}$	exponent in function for consumption of composite commodity c by household h
ρ_a^{va}	exponent in CES VA function for activity a
ρ_c^{x}	exponent in CET function for commodity c (top of nest)
$\rho_{f,a}^{2}$	exponent for level 2 production function factors that aggregate to f in activity a
σ_c^{q}	elasticity of substitution between supplies of domestic output and imports in Armington function for c (top of nest)

$\sigma_{c,h}^{qh}$	elasticity of substitution between commodities aggregated to composite commodity c for household h
σ_{a}^{va}	elasticity of substitution between factors in CES VA function of activity a
σ_{c}^{x}	elasticity of transformation between domestic sales and exports in CET function for c (top of nest)
$\sigma_{f,a}^{2}$	elasticity of substitution between level 2 production function factors that aggregate to f
φ_{c}^{q}	shift parameter in Armington function in which domestic sales and imports of commodity c (in C) are aggregated to composite supply (top of nest)
$\varphi_{c,h}^{qh}$	shift parameter for consumption of composite commodity c by household h
φ_{a}^{va}	shift parameter for CES VA function of activity a
φ_{c}^{x}	shift parameter in CET function for commodity c (top of nest)
$\varphi_{f,a}^{2}$	shift parameter for level 2 production function factors that aggregate to f in activity a

3.2. EQUATIONS

The equations are split into four blocks:

1) Production and factors;
2) Domestic and foreign trade;
3) Current accounts of domestic institutions; and
4) Investment, system constraints, and numéraire.

Each section of the presentation covers one block and has its equations stated in one table. In model simulations, it is possible to choose among alternative assumptions for (i) payments linking the government, domestic non-government institutions, and the rest of the world; and (ii) the equilibrating mechanisms (the closures) for macro balances, factor markets, and markets for exports and imports. In this presentation, we apply the following set of relatively simple assumptions:

- Government budget: The government balance is cleared by adjustments in government investment in the context of rule-based or exogenous levels for other government payments (including exogenous values for tax rates, quantities of government consumption, and foreign and domestic financing).
- Savings-investment: The level of domestically financed private investment is determined by the level of financing from domestic non-government institutions, for which the marginal

propensities to save are fixed. Government investment is financed as part of the government budget.

- Balance of payments: The balance is cleared by adjustments in the real exchange rate, which influence export and import quantities and values. Other items in the balance of payments (including transfers, foreign investment, and net foreign financing) are exogenous or determined by other rules.
- Markets for private capital: This factor is activity-specific (not mobile across activities); given this has activity-specific market-clearing wages.
- Markets for other factors: Other factors are mobile across relevant activities. However, for household activities, the labor employed may only come from the household that consumes the output.
- Foreign markets for exports and imports: Both world export and import prices are exogenous (i.e., the small country assumption applies).

3.2.1. PRODUCTION AND FACTORS

The equations in this block are found in Table 3.2.1. They cover the determination of production by sector, demands for factors and intermediates, TFP, factor wages (or rents), and factor incomes.

For convenience, the first equation (P1) defines the activity-specific wage (WFA), including taxes and, if relevant, a term for the rate of discrimination (which is treated differently from the other terms in the payment process). At the top level of the nested production structure, the activity levels (QA), which drive the level of commodity production by each activity, are a CES function of factor employment, scaled to account for the contribution of intermediate inputs (P2). Factor demands (QF) are a function of the parameters of the production function, wages, and the price of value added (i.e., the payment to factors per unit of the activity), in a setting where the producers maximize profits while taking prices and wages as given (P3).
Factors at this level (in the set F1) may be aggregations. Using CES functions also at the more disaggregated levels, a subset of the factors in F1 (in the set FNSAM, i.e. factors that are not in the SAM) may be aggregates of more disaggregated factors. In the set F2, with each element in F2 linked to a single element in F1 via the mapping MF1F2. Equation P4 shows the aggregation function while P5 shows the condition for an optimal cost-minimizing decision for employment of the more disaggregated factors in F2. In gender applications, the relevant elements may be aggregate labor (in F1) and labor disaggregated by gender (in F2). If labor also is disaggregated along some other dimension (like education), then the nesting structure may have additional levels.

TFP by activity is a function of an exogenous trend parameter, a scaling parameter (which typically is endogenous for the base simulation but otherwise exogenous), ratios between current and base-year government capital stocks, and openness to trade as the ratio between (a) the sum of real exports and imports; and (b) real GDP (P6). The latter ratio is defined in Table 3.2.2. In this equation, the impacts of government capital stocks and openness to trade are both captured by a constant-elasticity formulation.

Other variables related to production are determined by activity levels, other parameters, and prices. Intermediate demands (QINT) are a Leontief fixed-coefficient function of activity levels (P7). Likewise, commodity output levels (QX) are driven by activity levels multiplied by fixed yield coefficients (θ), summed over all relevant activities (P8). Depending on the values of the yield coefficients, any commodity may be produced by more than one activity and any activity may produce more than one commodity. The value-added price (PVA), which appeared above in the factor demand functions (P2), is defined as the price (or revenue) per unit of an activity (PA) net of activity taxes and the intermediate input cost per activity unit (P9). For any activity, PA is the product of yields and unit producer prices, summed over all outputs (P10).

The treatment of factor markets permits alternative assumptions with regard to wage determination and mobility between activities, including activities that are not part of GDP. In this mathematical statement, we assume that (i) private capital is fully employed and activity-specific (with endogenous allocations of private capital created by new investment); and (ii) that other factors (labor and natural resources, if any) are mobile. Other configurations are possible. Table 3.2.1 shows the treatment of the markets for factors other than private capital; the latter factor is treated in Table 3.2.4 given its links to investment and its special treatment of mobility.[12] Given that these factors are all mobile, the activity-specific wage term (WFDIST) is fixed (P10); this makes sure that, most importantly for labor, the model is able to reflect wage gaps for workers employed in different sectors. In most countries, such wage gaps are persistent, often with particularly low wages for agriculture.

The remaining equations in Table 3.2.1 define wages, employment, and factor incomes split into incomes from GDP and non-GDP activities. For non-GDP income, it may be more informative to refer to this as the value of time spent (since payments are rarely if ever made). To this end, equation P12 defines the time spent by labor from each household group (one or more) in non-GDP activities, drawing on a mapping that allocates each household activity to a unique household group (MHANGDP). To exemplify, each household group h may have a set of activities for male and female leisure as well as different household services, including child and elderly care; equation P12 defines the time allocated by male and female members of h to these activities. Equation P13 states that, for each household h, the time endowment of each labor

[12] The equations that permit variable capacity utilization for factors have been left out for conciseness since they are not (or at least less) relevant in the context of full accounting of discretionary time use for labor. In some contexts, they may still be relevant for capital and other non-labor factors.

type used in GDP activities is a residual: the full-time endowment net of what is used in non-GDP activities. For factors without non-GDP employment (in applications, typically all non-labor factors, including agricultural land), it simply states that the full supply is employed in GDP activities. Equations P14 and P15 define and impose equality between the total factor quantities supplied for GDP use (from the endowments of different institutions, including households) and demanded for GDP use by different activities. A flexible wage variable (WF) clears the market for each factor while maintaining relative wage differences (imposed by equation P11). Finally, equations P16 and P17 define the GDP and non-GDP factor incomes (YFGDP and YFNGDP), not including taxes. These variables reappear in the block of equations that covers institutional incomes. For factor incomes in GDP, this includes transfers from the rest of world and an adjustment to recycle potential implicit discrimination payments (which are considered in equations P1 and P5, assuming that the discrimination is against a disaggregated labor factor). According to the adjustment term, the payment is transferred from the factor that suffers from the discrimination (f') to another factor (f), on the basis of a mapping (MFDSC).

Table 3.2.1. Equations for production and factors

P1	$\mathrm{WFA}_{f,a,t} = \mathrm{WF}_{f,t} \cdot \mathrm{WFDIST}_{f,a,t}\left(1 + \mathrm{dsc}_{f,a,t}\right)\left(1 + \mathrm{TFA}_{f,a,t}\right)$	$f \in F$ $a \in A$ $t \in T$	Wages by activity (incl. tax)
P2	$\mathrm{QA}_{a,t} = \mathrm{TFP}_{a,t} \cdot \varphi_a^{va}\left(\sum_{f \in F} \delta_{f,a}^{va} \cdot \mathrm{QF}_{f,a,t}^{-\rho_a^{va}}\right)^{\frac{-1}{\rho_a^{va}}}$	$a \in A$ $t \in T$	Activity production
P3	$\mathrm{QF}_{f,a,t} = \left(\frac{\mathrm{PVA}_{a,t}}{\mathrm{WFA}_{f,a,t}}\right)^{\sigma_a^{va}} \left(\delta_{f,a}^{va}\right)^{\sigma_a^{va}} \left(\mathrm{TFP}_{a,t} \cdot \varphi_a^{va}\right)^{\sigma_a^{va}-1} \mathrm{QA}_{a,t}$	$f \in F1$ $a \in A$ $t \in T$	Factor demand -- functions at 1st (top) level
P4	$\mathrm{QF}_{f,a,t} = \varphi_{f,a}^{2}\left(\sum_{f\prime \in F2 \mid (f',f) MF2F1} \delta_{f\prime,a}^{2} \cdot \mathrm{QF}_{f',a,t}^{-\rho_{f,a}^{2}}\right)^{\frac{-1}{\rho_{f,a}^{2}}}$	$f \in F1$ $(\subset \mathrm{FNSAM})$ $a \in A$ $t \in T$	Factor demand -- aggregation of 2nd level
P5	$\mathrm{QF}_{f,a,t} = \left(\frac{\mathrm{WFA}_{f\prime,a,t}}{\mathrm{WFA}_{f,h,t}}\right)^{\sigma_{f\prime,a}^{2}} \left(\delta_{f,a}^{2}\right)^{\sigma_{f\prime,a}^{2}} \left(\varphi_{f\prime,a}^{2}\right)^{\sigma_{f\prime,a}^{2}-1} \mathrm{QF}_{f\prime,a,t}$	$f \in F2$ $f' \in F1$ (f, f') $\in MF2F1$ $a \in A$ $t \in T$	Factor demand -- functions at 2nd level

P6	$$\mathrm{TFP}_{a,t} = \mathrm{tfpb}_{a,t} \cdot \overline{\mathrm{TFPSCAL}_t} \cdot \prod_{f \in \mathrm{FCAPG}} \left(\frac{\mathrm{QFINS}_{gov,f,t}}{\mathrm{QFINS}^{00}_{gov,f}}\right)^{\eta^{tfp}_{a,f}} \cdot \left(\frac{\mathrm{TRDGDP}_t}{\mathrm{TRDGDP}^{00}}\right)^{\eta^{tfp}_{a,trdgdp}}$$	$a \in A, t \in T$	Total factor productivity
P7	$$\mathrm{QINT}_{c,a,t} = \mathrm{ica}_{c,a} \cdot \mathrm{QA}_{a,t}$$	$c \in C, a \in A$ $t \in T$	Intermediate demands
P8	$$\mathrm{QX}_{c,t} = \sum_{a \in A} \theta_{a,c} \cdot \mathrm{QA}_{a,t}$$	$c \in C, t \in T$	Output
P9	$$\mathrm{PVA}_{a,t} = \mathrm{PA}_{a,t}(1 - \mathrm{ta}_{a,t}) - \sum_{c \in C} \mathrm{PQD}_{c,a,t} \cdot \mathrm{ica}_{c,a}$$	$a \in A, t \in T$	Value-added price
P10	$$\mathrm{PA}_{a,t} = \sum_{c \in C} \theta_{a,c} \cdot \mathrm{PX}_{c,t}$$	$a \in A, t \in T$	Activity price
P11	$$\mathrm{WFDIST}_{f,a,t} = \mathrm{wfdistb}_{f,a}$$	$f \in FVA$ $f \notin FCAPNG$	Exogenous activity-specific wage term for mobile factors
P12	$$\mathrm{QFSNGDP}_{h,f,t} = \sum_{a \in \mathrm{ANGDP} \mid \mathrm{MHANGDP}(h,a)} \mathrm{QF}_{f,a,t}$$	h ∈ H f ∈ FLAB t ∈ T	Labor use in household activities (by household)
P13	$$\mathrm{QFSGDP}_{i,f,t} = \mathrm{QFINS}_{i,f,t} - \mathrm{QFSNGDP}_{i,f,t \mid i \in H, f \in \mathrm{FLAB}}$$	i ∈ INSD f ∈ FSAM t ∈ T	Factor supply to GDP activities (by institution)
P14	$$\mathrm{QFS}_{f,t} = \sum_{i \in \mathrm{INSD}} \mathrm{QFSGDP}_{i,f,t}$$	f ∈ FSAM t ∈ T	Total factor supply to GDP activities
P15	$$\mathrm{QFS}_{f,t} = \sum_{a \in \mathrm{AGDP}} \mathrm{QF}_{f,a,t}$$	f ∈ FSAM ∪ FNCAP t ∈ T	Market balance for GDP employment of non-capital factors
P16	$$\mathrm{YFGDP}_{f,t} = \sum_{a \in \mathrm{AGDP}} \mathrm{WF}_{f,t} \cdot \mathrm{WFDIST}_{f,a,t} \cdot \mathrm{QF}_{f,a,t} + \mathrm{TRNSFR}_{f,row,t} \cdot \mathrm{EXR}_t + \sum_{f' \in \mathrm{FSAM} \mid (f,f') \in \mathrm{MFDSC}} \sum_{a \in \mathrm{AGDP}} \mathrm{dsc}_{f',a,t} \cdot \mathrm{WF}_{f',t} \cdot \mathrm{WFDIST}_{f',a,t} \cdot \mathrm{QF}_{f',a,t}$$	f ∈ FSAM t ∈ T	GDP factor incomes

P17	$\mathrm{YFNGDP}_{hf,t} = \sum_{a \in \mathrm{ANGDP} \mid (a,h) \in \mathrm{MHANGDP}} \mathrm{WF}_{f,t} \cdot \mathrm{WFDIST}_{f,a,t} \cdot \mathrm{QF}_{f,a,t}$	$h \in H$ $f \in \mathrm{FLAB}$ $t \in T$	Non-GDP factor incomes

3.2.2. DOMESTIC AND AGGREGATE FOREIGN TRADE

Table 3.2.2 covers the allocation of domestic commodity demands between imports and domestic output and the allocation of domestic output between exports and domestic sales. Equations T1-T3 are related to prices. In T1, the export price received by producers, PE, is defined as the world export price, transformed into domestic currency via the exchange rate and adjusted for export taxes and the transactions (trade and transport) cost per unit of exports; the unit transactions cost is defined as the product of an input coefficient (ice) and the input price, summed over all inputs. In analogous fashion, equation T2 defines the domestic currency import price for demanders, PM, on the basis of the world import price, the exchange rate, and import tariffs, in this case with the unit transactions cost added to the price. In both equations, it is assumed that the modeled economy is small; thus, world prices for exports and imports (pwe and pwm) are exogenous. Equation T3 links the demander and supplier prices for domestic output sold domestically, PDD and PDS: the demander price is defined as the supplier price plus the transactions cost per unit of domestically sold output. As will be discussed below, either of these prices can be seen as the market-clearing price for this category of outputs (cf. equation S3).
Commodity demand, QQ, is a CES aggregation of imports and domestic purchases, called an Armington function after its originator (T4). QQ is referred to as a "composite" demand given that it is met from different sources. Equation T5 defines the composite demands for commodities that (in contrast to those covered by T4) do not have both imports and domestic purchases.

For commodities with both sources, domestic demanders are assumed to minimize the cost of any composite demand quantity subject to the Armington function and relative prices. The first-order conditions (FOCs) are made up of the Armington function itself (T4), and an equation that specifies the optimal demand ratio (QM/QD) as a function of the ratio between the prices of domestic output and imports (PDD/PM) (T6). The composite price PQS is implicitly defined by T7 given that the other variables in this equation are determined by other relationships. At the composite commodity level, a distinction is made between PQS and PQD. As shown by T8, the distinction is that PQD (the price paid by domestic demanders) is adjusted to account for sales taxes, value-added taxes, and subsidies. Given that both value-added taxes and subsidies always or often have different rates per demander category, PQD is disaggregated along this additional dimension (captured by the index d).

Turning to the production side, a constant-elasticity-of-transformation (CET) function (T9) defines the frontier for allocations of domestic output (QX) between exports and domestic sales (QE and QD, respectively) for outputs that according to base data have non-zero values for both destinations. Equation T10 defines the equivalent of this transformation for outputs with only domestic sales or only exports.

For outputs with both destinations, producers are assumed to maximize the revenue of any output quantity subject to the CET function and relative prices. The FOCs are made up of the CET function and an equation that specifies the optimal supply ratio (QE/QD) as a function of the ratio between the prices of exports and domestic sales (PE/PDS) (T11). The average producer output price, PX, is defined as a weighted average of the prices received for domestic sales and exports (T12). (In section 3.2.1, PX influences production decisions and revenues.) The demand for trade and transport services is a function of real domestic and foreign trade volumes, using a fixed-coefficient formulation (T13). The final two equations in this block define the real trade-GDP ratio and real GDP, which is the denominator in this ratio (T14 and T15).

Table 3.2.2. Equations for domestic and aggregate foreign trade

T1	$\mathrm{PE}_{c,t} = (1 - \mathrm{te}_{c,t})\mathrm{EXR}_t \cdot \mathrm{pwe}_{c,t} - \sum_{c\prime \in \mathrm{CT}} \sum_{\mathrm{tace} \in \mathrm{TACE}} \mathrm{PQD}_{c\prime,\mathrm{tace},t} \mathrm{ice}_{c\prime,c}$	$c \in \mathrm{CE}$ $t \in \mathrm{T}$	Export price
T2	$\mathrm{PM}_{c,t} = (1 + \mathrm{tm}_{c,t})\mathrm{EXR}_t \cdot \mathrm{pwm}_{c,t} + \sum_{c\prime \in \mathrm{CT}} \sum_{\mathrm{tacm} \in \mathrm{TACM}} \mathrm{PQD}_{c,\mathrm{tacm},t} \cdot \mathrm{icm}_{c\prime,c}$	$c \in \mathrm{CM}$ $t \in \mathrm{T}$	Import price
T3	$\mathrm{PDD}_{c,t} = \mathrm{PDS}_{c,t} + \sum_{c\prime \in \mathrm{CT}} \sum_{\mathrm{tacd} \in \mathrm{TACD}} \mathrm{PQD}_{c,\mathrm{tacd},t} \mathrm{icd}_{c\prime,c}$	$c \in \mathrm{C}$ $t \in \mathrm{T}$	Domestic demand price for domestic output
T4	$\mathrm{QQ}_{c,t} = \varphi_c^{\mathrm{q}} \left(\delta_c^{\mathrm{m}} \cdot \mathrm{QM}_{c,t}^{-\rho_c^{\mathrm{q}}} + \delta_c^{\mathrm{dd}} \cdot \mathrm{QD}_{c,t}^{-\rho_c^{\mathrm{q}}} \right)^{-\frac{1}{\rho_c^{\mathrm{q}}}}$	$c \in \mathrm{CM} \cap \mathrm{CD}$ $t \in \mathrm{T}$	Composite demand if use of imports and domestic output
T5	$\mathrm{QQ}_{c,t} = \mathrm{QM}_{c,t} + \mathrm{QD}_{c,t}$	$(c \in \mathrm{CM} \cap c \notin \mathrm{CD}) \cup (c \in \mathrm{CD} \cap c \notin \mathrm{CM})$, $t \in \mathrm{T}$	Composite demand if not use of both imports and

			domestic output
T6	$\frac{QM_{c,t}}{QD_{c,t}} = \left(\frac{PDD_{c,t}}{PM_{c,t}} \frac{\delta_c^m}{\delta_c^{dd}}\right)^{\frac{1}{1+\rho_c^q}}$	$c \in CM \cap CD$ $t \in T$	Import-domestic demand ratio
T7	$PQS_{c,t} \cdot QQ_{c,t} = \left(PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t}\right)$	$c \in C$ $t \in T$	Composite demand price
T8	$PQD_{c,d,t} = PQS_{c,t}(1 + tq_{c,t})(1 - sub_{c,d,t})(1 + tva_{c,d,t})$	$c \in C, d \in D$ $t \in T$	Adjusted composite demand price
T9	$QX_{c,t} = \varphi_c^x \left(\delta_c^e \cdot QE_{c,t}^{\rho_c^x} + \delta_c^{ds} \cdot QD_{c,t}^{\rho_c^x}\right)^{\frac{1}{\rho_c^x}}$	$c \in CE \cap CD$ $t \in T$	Output transfor-mation if both exports and domestic sales
T10	$QX_{c,t} = QE_{c,t} + QD_{c,t}$	$(c \in CE \cap c \notin CD) \cup (c \in CD \cap c \notin CE)$, $t \in T$	Output transformation if not both exports and domestic sales
T11	$\frac{QE_{c,t}}{QD_{c,t}} = \left(\frac{PE_{c,t}}{PDS_{c,t}} \frac{\delta_c^{ds}}{\delta_c^e}\right)^{\frac{1}{\rho_c^x - 1}}$	$c \in CE \cap CD$ $t \in T$	Export-domestic sales ratio
T12	$PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t}$	$c \in C$ $t \in T$	Producer output price
T13	$QT_{c,t} = \sum_{c' \in C} \left(icm_{c,c'} \cdot QM_{c',t} + ice_{c,c'} \cdot QE_{c',t} + icd_{c,c'} \cdot QD_{c',t}\right)$	$c \in CT$ $t \in T$	Trade and transport margin demands
T14	$\frac{TRDGDP_t = \sum_{c \in C} EXR^{00} \cdot pwe_c^{00} \cdot QE_{c,t} + \sum_{c \in C} EXR^{00} \cdot pwm_c^{00} \cdot QM_{c,t}}{RGDPMP_t}$	$t \in T$	Real trade-GDP ratio
T15	$RGDPMP_t = \sum PQD_{c,h}^{00} \cdot QH_{c,h,t}$	$t \in T$	Real GDP at market prices

	$+\sum_{c\in C} PQD^{00}_{c,dstk} \cdot \sum_{i\in INS} qdstk_{c,i,t}$ $+\sum_{c\in C} PQD^{00}_{c,gov} \cdot QG_{c,t}$ $+\sum_{c\in C} EXR^{00} \cdot pwe^{00}_{c} \cdot QE_{c,t} - \sum_{c\in C} EXR^{00} \cdot pwm^{00}_{c} \cdot QM_{c,t}$		

3.2.3. CURRENT PAYMENTS BY DOMESTIC INSTITUTIONS

This equation block explains payments that are part of the current accounts of domestic institutions (i.e. current incomes and spending for households, the government and enterprises). In the model and its database, it is necessary to include at least one household and in practice, models applied to real-world countries invariably have a government. Enterprises are optional since the payments they receive and make could be attributed to the owners of the enterprises. Even though the model and the database can handle multiple representative households, this mathematical statement assumes for simplicity that there is only one household to avoid the inclusion of equations that are related to the dynamics of household payments in the context of demographic change. The sets for institutions distinguish between INSD (all domestic institutions), INSDNG (all non-government domestic institutions, i.e. households and enterprises), and H (households, which may include "non-profit institutions in service of households").

Enterprises differ from households in that they do not consume.
On the income side, the shares of domestic institutions in GDP factor incomes, SHIF, are defined on the basis of their GDP employment shares (I1). Employment is defined in Section 3.2.1. The factor incomes of domestic institutions, YIF, are defined as non-GDP factor incomes (defined by institution in Section 3.2.1) plus GDP factor incomes, which depend on these institutional shares, total GDP factor incomes (YFGDP) net of direct taxes, and exogenous payments of factor incomes to the outside world (I2). Using this information, the total incomes of domestic non-government institutions, YI, are the sum of factor incomes, transfers from the government (indexed to the numéraire, in this case the CPI), transfers from abroad, and transfers from other institutions in INSDNG (I3). Government incomes are defined in a separate equation.

The values for consumption and transfer spending by domestic non-government institutions are defined after deducting payments for direct taxes and savings. The mathematical statement treats direct tax rates as exogenous (policy-determined) but demonstrates alternative treatments for savings rates. The marginal propensity to save, MPS, is the product of an institution-specific

rate that may change over time and a scaling parameter (I4). If the latter is flexible, then total savings are adjusted endogenously in the context of restrictions on the total quantity or value of private investment financed by domestic non-government institutions. Here, the scaling parameter is fixed, meaning that investment spending must be flexible (this is discussed in Section 3.2.4.). Institution-specific savings values, SAV, are a linear function of MPS and income net of direct taxes, with an optional non-zero intercept, which is indexed to the numéraire (I5). The presence of an intercept is essential when base-year data indicate that some household groups have negative savings. Without a separate (negative) intercept and the related assumption that marginal and average savings rates differ, higher incomes would in this setting reduce savings further below zero (since the savings rate would be negative). Transfers from institutions in INSDNG to other institutions (in INS), TRII, are fixed shares of their incomes net of direct taxes and savings (I6). For households, consumption spending, EH, is defined as income net of direct taxes, savings, and transfers to other institutions (I7).

Household consumption demand has a nested, two-level structure, with linear expenditure system (LES) functions at the top and CES functions at the bottom. The nesting was needed to define households service demands for different types of care and other services as aggregates of services provided by the household itself and the market. Equation I8 defines aggregate demands (QH defined over the set C1) as an LES function of population, prices, and total spending (EH). These demand functions are derived from the maximization of a Stone-Geary utility function subject to total spending and prices.[13] Equations I9 and I10 cover the second level of the household consumption nest: (a) I9 defines the quantities of top-level commodities (household services) that are aggregates (elements in C1 and CNSAM, i.e. not in the SAM) as a CES function of the quantities of the disaggregated commodities that are linked to the aggregate (via the mapping MC2C1); and (b) I10 defines the optimal cost-minimizing allocation of the demands between disaggregated commodities (in C2, all of which are in the SAM).

The remaining equations in this block define current government receipts and spending. Government receipts, YG, are the sum of relatively disaggregated tax revenue flows, domestic and foreign transfers, and factor incomes (I11). Whether a given tax is part of an application depends on the database. To make the mathematical statement more digestible, equations I12, I13, and I15 define separate variables for tax revenues with relatively complex expressions: direct taxes on institutions, value-added taxes, and activity-specific factor taxes, respectively. For the latter, equation I14 defines the tax rates as a function of base rates and a scaling variable (TFASCAL) that here is exogenous (given that tax rates are fixed).[14] Domestic transfers are exogenous and indexed to the numéraire; foreign transfers are exogenous in FCU.

[13] It is referred to as a linear expenditure system (LES) since spending on any commodity (the product of price and quantity) is a linear function of EH. This is evident if one multiplies both sides of I8 by the price variable (PQD).
[14] In other settings, TFASCAL could be endogenous. For example, if the rates are defined endogenously to achieve a target GDP share for this revenue flow. However, this requires additional variables and equations that, for the

Current government spending, EG, is the sum of spending on consumption, domestic transfers, transfers abroad, and subsidies (I16). The quantities of government consumption, QG, are defined on the basis of a trend term (qgb) that may be scaled selectively by commodity and time period (I17). The impact of a given value for the scaling variable QGSCAL depends on the level of the parameter qg01. As indicated by its name, we propose that it be set at values between 0 and 1, among other things meaning that commodities with a value of zero are not affected by any scaling of government consumption. In the current mathematical statement, QGSCAL is exogenous; if it were endogenous, it could be used to clear the government budget. Subsidy spending, SUBCT, is also defined in a separate equation (I18); subsidy rates are similar to VAT rates in that they are disaggregated by commodity, demander, and time period.

Table 3.2.3. Equations for current payments involving domestic institutions

I1	$$SHIF_{i,f,t} = \frac{QFSGDP_{i,f,t}}{\sum_{i\prime \in INSD} QFSGDP_{i\prime,f,t}}$$	$i \in INSD$ $f \in FSAM$ $t \in T$	Shares of GDP factor incomes to domestic institutions
I2	$$YIF_{i,f,t} = YFNGDP_{i,f,t_{i\in H}} + SHIF_{i,f,t}\left(YFGDP_{f,t}\left(1 - TF_{f,t}\right) - TRNSFR_{row,f,t} \cdot EXR_t\right)$$	$i \in INSD$ $f \in FSAM$ $t \in T$	Non-GDP + GDP factor incomes to domestic institutions)
I3	$$YI_{i,t} = \sum_{f\in FVA} YIF_{i,f,t} + trnsfr_{i,gov,t} \cdot \overline{CPI_t} + trnsfr_{i,row,t} \cdot EXR_t + \sum_{i\prime \in INSDNG} TRII_{i,i\prime,t}$$	$i \in INSDNG$ $t \in T$	Domestic non-gov't institution income
I4	$$MPS_{i,t} = mpsb_{i,t} \cdot \overline{MPSSCAL_t}$$	$i \in INSDNG$ $t \in T$	Marginal propensity to save
I5	$$SAV_{i,t} = \alpha_{i,t}^{sav} \cdot \overline{CPI_t} + MPS_{i,t}\left(YI_{i,t} - YTAXDIRI_{i,t}\right)$$	$i \in INSDNG$ $t \in T$	Domestic non-gov't institution savings
I6	$$TRII_{i\prime,i,t} = shii_{i\prime,i}\left(YI_{i,t} - YTAXDIRI_{i,t} - SAV_{i,t}\right)$$	$i \in INSDNG$ $i' \in INS$ $t \in T$	Institutional transfers
I7	$$EH_{h,t} = YI_{h,t} - YTAXDIRI_{h,t} - SAV_{h,t} - \sum_{i\in INS} TRII_{i,h,t}$$	$h \in H$ $t \in T$	Household consumption expenditure
I8	$$QH_{c,h,t} = \gamma_{c,h,t} \cdot pop_{h,t} + \frac{\beta_{c,h}\left(EH_{h,t} - \sum_{c\prime \in C1} PQD_{c\prime,h,t} \cdot \gamma_{c\prime,h,t} \cdot pop_{h,t}\right)}{PQD_{c,h,t}}$$	$c \in C1$ $h \in H$ $t \in T$	Household consumption -- demand at 1st (top) level

sake of brevity, are not included in this statement but are available in the code. Similar scaling variables also exist for other tax rates that, for simplicity, here are presented as parameters (tf, ta, tq, te, and tm).

I9	$$QH_{c,h,t} = \varphi_{c,h}^{qh}\left(\sum_{c\prime\in C2_{\mid MC2C1(c',c)}} \delta_{c\prime,h}^{qh}\cdot QH_{c\prime,h,t}^{-\rho_{c,h}^{qh}}\right)^{\frac{-1}{\rho_{c,h}^{qh}}}$$	$c \in CNSAM\ (\subset C1)$ $h \in H$ $t \in T$	Household consumption demand - -aggregation of 2nd level
I10	$$QH_{c,h,t} = \left(\frac{PQD_{c\prime,h,t}}{PQD_{c,h,t}}\right)^{\sigma_{c\prime,h}^{qh}}\left(\delta_{c,h}^{qh}\right)^{\sigma_{c\prime,h}^{qh}}\left(\varphi_{c\prime,h}^{qh}\right)^{\sigma_{c\prime,h}^{qh}-1} QH_{c\prime,h,t}$$	$c \in C2$ $c' \in C1$ (c, c') $\in MC2C1$ $h \in H$ $t \in T$	Household consumption demand - - functions at 2nd level
I11	$$YG_t = \sum_{i\in INSDNG} YTAXDIRI_{i,t} + \sum_{f\in F}\overline{TF}_{f,t}\cdot YFGDP_{f,t} + \sum_{c\in C} tq_{c,t}\cdot PQS_{c,t}\cdot QQ_{c,t} + YTAXVA_t + YTAXFA_t + \sum_{a\in A} ta_{a,t}\cdot PA_{a,t}\cdot QA_{a,t} + \sum_{c\in C} te_{c,t}\cdot pwe_{c,t}\cdot QE_{c,t}\cdot EXR_t + \sum_{c\in C} tm_{c,t}\cdot pwm_{c,t}\cdot QM_{c,t}\cdot EXR_t + trnsfr_{gov,row,t}\cdot EXR_t + \sum_{i\in INSDNG} TRII_{gov,i,t} + \sum_{f\in F} YIF_{gov,f,t}$$	$t \in T$	Government current receipts
I12	$$YTAXDIRI_{i,t} = ty_{i,t}\left(YI_{i,t} - \sum_{f\in FLAB} YFNGDP_{i,f,t}\right)$$	$i \in INSDNG$ $t \in T$	Government direct tax revenue from domestic institution

I13	$$\begin{aligned}YTAXVA_t = &\sum_{c\in C}\sum_{a\in A}(1-sub_{c,a,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\cdot tva_{c,a,t}\\ &\cdot QINT_{c,a,t}\\ &+\sum_{c\in C}\sum_{h\in H}(1-sub_{c,h,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\\ &\cdot tva_{c,a,t}\cdot QH_{c,h,t}\\ &+\sum_{c\in C}(1-sub_{c,gov,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\\ &\cdot tva_{c,gov,t}\cdot QG_{c,t}\\ &+\sum_{c\in C}\sum_{f\in FCAP}\Big((1-sub_{c,f,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\\ &\cdot tva_{c,f,t}\\ &\cdot capcomp_{c,f}\sum_{i\in INS}DKINS_{i,f,t}\Big)\\ &+\sum_{c\in C}\sum_{i\in INS}(1-sub_{c,dstk,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\\ &\cdot tva_{c,dstk,t}\cdot qdstk_{c,i,t}\\ &+\sum_{c\in C}\sum_{c'\in CDIS}(1-sub_{c,tacm,t})\cdot PQS_{c,t}\\ &\cdot(1+tq_{c,t})\cdot tva_{c,tacm,t}\cdot icm_{c,c',r}\cdot QMR_{c',r,t}\\ &+\sum_{c\in C}\sum_{c'\in CDIS}(1-sub_{c,tace,t})\cdot PQS_{c,t}\\ &\cdot(1+tq_{c,t})\cdot tva_{c,tace,t}\cdot ice_{c,c',r}\cdot QER_{c',r,t}\\ &+\sum_{c\in C}\sum_{c'\in C}(1-sub_{c,tacd,t})\cdot PQS_{c,t}\cdot(1+tq_{c,t})\\ &\cdot tva_{c,tacd,t}\cdot icd_{c,c'}\cdot QD_{c',r,t}\end{aligned}$$	$t \in T$	VAT revenue
I14	$$TFA_{f,a,t} = tfab_{f,a,t}\cdot\overline{TFASCAL_t}$$	f ∈ FSAM a ∈ A t ∈ T	
I15	$$YTAXFA_t = \sum_{f\in FSAM} TFA_{f,a,t}\cdot WF_{f,t}\cdot WFDIST_{f,a,t}\cdot QF_{f,a,t}$$	t ∈ T	
I16	$$EG_t = \sum_{c\in C} PQD_{c,gov,t}\cdot QG_{c,t} + \sum_{i\in INSDNG} trnsfr_{i,gov,t}\cdot\overline{CPI_t} + trnsfr_{row,gov,t}\cdot EXR_t + SUBCT_t$$	t ∈ T	Government expenditure
I17	$$QG_{c,t} = qgb_{c,t}\left(1+qg01_{c,t}\cdot\overline{QGSCAL_t}\right)$$	c ∈ C t ∈ T	Government consumption
I18	$$SUBCT_t =$$	t ∈ T	Commodity subsidy

	$\sum_{c\in C}\sum_{a\in A} \text{sub}_{c,a,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{QINT}_{c,a,t}$ $+\sum_{c\in C}\sum_{h\in H} \text{sub}_{c,h,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{QH}_{c,h,t}$ $+\sum_{c\in C} \text{sub}_{c,gov,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{QG}_{c,t}$ $+\sum_{c\in C}\sum_{f\in FCAP} \text{sub}_{c,f,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t})$ $\cdot \text{capcomp}_{c,f} \sum_{i\in INS} \text{DKINS}_{i,f,t}$ $+\sum_{c\in C}\sum_{i\in INS} \text{sub}_{c,dstk,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{qdstk}_{c,i,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} \text{sub}_{c,tacm,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{icm}_{c,c',r}$ $\cdot \text{QMR}_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in CDIS} \text{sub}_{c,tace,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{ice}_{c,c',r} \cdot \text{QER}_{c',r,t}$ $+\sum_{c\in C}\sum_{c'\in C} \text{sub}_{c,tacd,t} \cdot \text{PQS}_{c,t} \cdot (1+\text{tq}_{c,t}) \cdot \text{icd}_{c,c'} \cdot \text{QD}_{c',r,t}$		

3.2.4. INVESTMENT, SYSTEM CONSTRAINTS, AND NUMÉRAIRE

This block covers the system constraints that have not yet been covered: the constraints related to investment financing; the (domestic) markets for private capital and commodities; and the balance of payments.[15] In Sections 3.2.1 and 3.2.3, we covered the markets for other factors and the government balance, respectively. In addition, we here specify the numéraire, needed for CGE models like GEM-Care. The bulk of the equations cover how new capital (from investment) feeds into the economy.

For the government, investment spending (or gross capital formation), INVG, is defined as the sum of government savings (the difference between current receipts and spending), domestic net financing (indexed to the numéraire), and foreign net financing (exogenous in FCU) (S1). The

[15] The term "system constraints" refers to constraints that are not perceived by individual actors, such as household budget constraints, but which the economic system nevertheless must respect. For example, a labor market constraint that says that the quantity employed equals the quantity that is available.

sum of the two financing terms is the government primary deficit. These two terms are referred to as net financing items since they represent the difference between new borrowing and interest payments; the latter do not appear explicitly in the model.[16] For domestic non-government institutions, the corresponding variable, INV, is the sum of own savings and net financing from abroad minus claims on investment funding to finance the government and add to foreign reserves (S2). Before translating investment spending into quantities of new capital, it is necessary to specify prices. In equation S3, the unit prices of new capital stocks, PK, are defined as the product of the price of commodity c and the matrix of capital composition coefficients (showing the quantities of commodities c used as inputs per unit of any new capital stock f) summed over all c.

The next three equations define investment quantities by destination (type of capital stock), by government and non-government institutions, and investment quantities by source (the use of commodity inputs in the production of new capital). For the government, the quantity of new capital stock f, $DKINS_{gov,f,t}$, is defined as investment spending net of spending on new inventory (gross fixed capital formation or GFCF), multiplied by the spending share for f, and divided by PK to transform into stock quantities (S4). In the corresponding equation for non-government investment (S5), GFCF is the sum of: (a) investment net of stock change spending for institutions in INSDNG; and (b) foreign investment (the value of which is exogenous in FCU). GFCF is allocated across different capital stocks (if more than one) in fixed spending shares and transformed into quantities of new capital by dividing by PK.[17] Final investment demands (i.e. investment quantities defined by the source of inputs into the construction of new capital), QINV, are defined as the product of the capital composition matrix and investment by capital stock, summed over all capital stocks (S6).

For any capital stock, the endowments held by domestic institutions (government and non-government), QFINS, are defined as the sum of: (a) the stock held in the previous year net of depreciation; and (b) new investment in the previous year (S7). For the stocks of other factors, endowments are defined exogenously (S8). The values for endowments were used in Section 3.2.1 to define the supply sides of markets for non-capital factors and, in Section 3.2.3, to define the distributional shares for factor incomes.

[16] Given simulation results for government and private net financing (domestic and foreign) and with the help of additional data on initial debt stocks, and real interest rates by time period, debt stocks are defined in post-calculations (after the model is solved) and reported in different ways (among other things as shares of GDP and export values).

[17] From our experience, in most applications of these models, it is preferable to have one type of private capital with constrained mobility. However, the option of multiple stocks may be useful if, for example, the model should mimic the allocation of foreign investment to specific sectors like mining.

The allocation of private capital stocks across activities responds to relative capital rents.[18] As an input to the formulation used, the average wage of private capital stock f, WFAVG, is defined as total rent to f divided by total employment of f (S9). In equation S10, the allocation of new private capital stock f to an activity a, DKA, is defined as the product of: (a) an allocation based on current activity shares (i.e. total new investment in f times the current share of a in the use of f); and (b) an adjustment term that is above (below) unity if the wage of capital stock f in a is above (below) the economy-wide average, assuming a positive value for the parameter κ (Greek kappa; κ ≥ 0). κ plays a crucial role in this formulation: the higher its value, the stronger the sensitivity of the allocation of new capital to differences in capital rents; if it is zero, the allocation of stock f does not change over time; and if it is too high, capital rents may oscillate.[19]

Total employment of capital stock f in activity a in period t, $QF_{f,a,t}$, is defined as the stock installed in t-1, $QF_{f,a,t-1}$, net of depreciation, plus the quantity of new investment in stock f in t-1 allocated to a, $DKA_{f,a,t-1}$ (S11). This last equation may be seen as defining a set of activity-specific markets for capital stock f in which the quantity supplied (the right-hand side) is fixed within any period t (determined by past decisions) while the quantity demanded (the left-hand side) is determined by profit-maximization (cf. Section 3.2.1). A wage variable defined over f and a is needed to clear this market. Accordingly, among the two wage variables that apply to any factor ($WF_{f,t}$ and $WFDIST_{f,a,t}$), equation S12 fixes the economywide variable WF while leaving the activity specific variable WFDIST flexible. The simulated values for the product of the two variables show the scarcity value of private capital stocks by activity. In sum, for private capital, it is assumed that installed stocks cannot be reallocated while the analyst controls the extent to which the allocation of new capital will shift toward sectors with relatively high capital rents.

For each domestic commodity, the demand side is now complete. Equation S13 defines total composite demand for any commodity, QQ, as the sum of consumption, investment (fixed capital formation and stock changes), intermediate demands, and demands for trade and transportation services (due to domestic and foreign trade). As specified in Section 3.2.2, these demands generate demands for domestic output and/or imports. The markets for domestic output sold domestically are cleared by the linked variables PDD and PDS. To exemplify for the case of excess demand, increases in both price variables would simultaneously reduce domestic demands

[18] This approach that is presented may have been first developed in Dervis et al. (1982, pp. 175-178). Our treatment deviates in one respect: for simplicity, we use capital rents by activity instead of profit rates.

[19] With reference to S10, by definition, $\sum_{a\in A} DKA_{f,a,t} = \sum_{i\in INSNG} DKINS_{f,i,t}$ for $f \in FCAPNG$ and $t \in T$. This can be shown as follows: For simplicity, replacing $\sum_{i\in INSNG} DKINS_{f,i,t}$ by DKI and $WF_{f,t} \cdot WFDIST_{f,a,t}$ by WFA_a, suppressing remaining f and t subscripts, and noting the definition of the average wage, $DKA_a = DKI \cdot \frac{QF_a}{\sum_{a'\in A} QF_{a'}} \cdot \left(1+\kappa\left(\frac{WFA_a}{\overline{WF}}-1\right)\right)$; $\sum_{a\in A} DKA_a = DKI \cdot \sum_{a\in A}\left(\frac{QF_a}{\sum_{a'\in A} QF_{a'}} \cdot \left(1+\kappa\left(\frac{WFA_a}{\overline{WF}}-1\right)\right)\right)$

$$= DKI\left(\frac{\sum_{a\in A} QF_a}{\sum_{a'\in A} QF_{a'}} + \kappa \cdot \sum_{a\in A}\left(\frac{WFA_a}{\overline{WF}}\frac{QF_a}{\sum_{a'\in A} QF_{a'}}\right) - \frac{\sum_{a\in A} QF_a}{\sum_{a'\in A} QF_{a'}}\right) = DKI\left(1+\kappa\left(\frac{\overline{WF}}{\overline{WF}}-1\right)\right) = DKI$$

for domestic outputs and increase the quantities of output sold domestically (raising the total output level by raising profitability and raising the share of output sold domestically).

The statement of investment financing completes the flows in the balance of payments, which is expressed in FCU. Equations S14 and S15 state the current and capital accounts, respectively, with foreign savings, SAVF, as the linking variable. In the current account balance, inflows are due to exports and transfers from abroad while outflows are caused by imports, transfers from domestic non-government institutions, and factor incomes. The variable SAVF measures the current-account deficit; if outflows (the right-hand side) are larger (smaller) than inflows (the left-hand side), foreign savings are positive (negative).[20] In the capital account balance, the current account deficit is financed by net foreign financing to government and non-government institutions and foreign investment, net of increases of foreign reserves. By influencing export and import quantities in opposite directions, raising or reducing the trade balance in FCU, adjustments in the exchange rate, EXR, clear the balance of payments, making sure that the level of foreign savings matches the level that is financed on the right-hand side of the capital account. As a manifestation of Walras' law, in a CGE model like the one presented above, one equation should be removed to assure equality between the number of variables and independent equations. It is possible to check that the omitted equation holds in a post-calculation. Here we opt for the alternative of instead inserting one variable, named WALRAS, into one equation. Hence, the presence of WALRAS in the capital account of the balance of payments. In the absence of errors, the solution value for WALRAS should be very close to zero.

Finally, a well-specified CGE model like GEM-Care is homogeneous of degree zero in prices, meaning that only relative prices matter and that if one set of relative prices solves the model, then any multiple of this set of prices also solves the model (scaling all domestic prices and payments) without any influence on quantities. To anchor the price level, a price or price index referred to as the numéraire needs to be fixed, with the consequence that all other prices are measured relative to this numéraire. In this mathematical statement, the consumer price index, CPI, is the numéraire. Here, equation S16 defines the CPI, which is fixed on the basis of the base-year weights of household consumption payments by commodity and household type in total household consumption.[21]

[20] The variable *SAVF* deviates from the definition of foreign savings given that interest payments are an implicit part of the capital account instead of being part of the current account. The variable *SAVF* could more accurately be referred to as the primary deficit of the nation.

[21] As an alternative to the CPI, the domestic producer price index (DPI) may serve as numéraire. In addition, it is often used as the denominator in the definition of the price-level-deflated (PLD) real exchange rate (REXR). Algebraically, with time subscripts omitted, $DPI = \sum_{c \in C} PDS_c \cdot dwts_c$ and $REXR = \frac{EXR}{DPI}$.

Table 3.2.4. Equations for investment, system constraints, and numéraire

S1	$\mathrm{INVG}_t = (\mathrm{YG}_t - \mathrm{EG}_t) + \mathrm{ndfg}_t \cdot \overline{\mathrm{CPI}}_t + \mathrm{nff}_{gov,t} \cdot \mathrm{EXR}_t$	$t \in T$	Gov't primary deficit, investment value, and financing
S2	$\mathrm{INV}_{i,t} = \mathrm{SAV}_{i,t} + \mathrm{nff}_{i,t} \cdot \mathrm{EXR}_t - \left(\frac{\mathrm{SAV}_{i,t}}{\sum_{i' \in \mathrm{INSDNG}} \mathrm{SAV}_{i',t}}\right)\left(\mathrm{ndfg}_t \cdot \overline{\mathrm{CPI}}_t + \mathrm{drf}_t \cdot \mathrm{EXR}_t\right)$	$i \in \mathrm{INSDNG}$ $t \in T$	Non-gov't investment value and its financing
S3	$\mathrm{PK}_{f,t} = \sum_{c \in C} \mathrm{PQD}_{c,f,t} \cdot \mathrm{capcomp}_{c,f}$	$f \in \mathrm{FCAP}$ $t \in T$	Price of new capital
S4	$\mathrm{DKINS}_{gov,f,t} = \frac{\mathrm{invshr}_{f,gov,t} \cdot (\mathrm{INVG}_t - \sum_{c \in C} \mathrm{PQD}_{c,gov,t} \cdot \mathrm{qdstk}_{c,gov,t})}{\mathrm{PK}_{f,t}}$	$f \in \mathrm{FCAPG}$ $t \in T$	Gov't investment by government capital stock
S5	$\mathrm{DKINS}_{i,f,t} = \frac{\mathrm{invshr}_{f,i,t}}{\mathrm{PK}_{f,t}} \cdot \left(\left(\mathrm{INV}_{i,t} - \sum_{c \in C} \mathrm{PQD}_{c,i,t} \cdot \mathrm{qdstk}_{c,i,t}\right)\Big\vert_{i \in \mathrm{INSDNG}} + \left(\mathrm{invf}_{i,t} \cdot \mathrm{EXR}_t\right)\Big\vert_{i \in \mathrm{INSROW}} \right)$	$f \in \mathrm{FCAPNG}$ $i \in \mathrm{INSNG}$ $t \in T$	Non-gov't investment by private capital stock
S6	$\mathrm{QINV}_{c,t} = \sum_{i \in \mathrm{INS}} \sum_{f \in \mathrm{FCAP}} \mathrm{capcomp}_{c,f} \cdot \mathrm{DKINS}_{i,f,t}$	$c \in C$ $t \in T$	Real investment demand (by source)
S7	$\mathrm{QFINS}_{i,f,t} = \mathrm{QFINS}_{i,f,t-1}(1 - \mathrm{depr}_{f,t-1}) + \mathrm{DKINS}_{i,f,t-1}$	$i \in \mathrm{INSD}$ $f \in \mathrm{FCAP}$ $t \in T$ $t \notin \mathrm{TMIN}$	Accumulation of capital by domestic institutions
S8	$\mathrm{QFINS}_{i,f,t} = \mathrm{qfinsb}_{i,f,t}$	$i \in \mathrm{INSD}$ $f \in \mathrm{FOTH}$ $t \in T$	Exogenous institutional endowments for other factors
S9	$\mathrm{WFAVG}_{f,t} = \frac{\sum_{a \in A} \mathrm{WF}_{f,a,t} \cdot \mathrm{WFDIST}_{f,a,t} \cdot \mathrm{QF}_{f,a,t}}{\sum_{a \in A} \mathrm{QF}_{f,a,t}}$	$f \in \mathrm{FCAPNG}$ $t \in T$	Average wage (rent) by private capital stock
S10	$\mathrm{DKA}_{f,a,t} = \left(\sum_{i \in \mathrm{INSNG}} \mathrm{DKINS}_{f,i,t}\right)\left(\frac{\mathrm{QF}_{f,a,t}}{\sum_{a' \in A} \mathrm{QF}_{f,a',t}}\right) \cdot \left(1 + \kappa_f \left(\frac{\mathrm{WF}_{f,t} \cdot \mathrm{WFDIST}_{f,a,t}}{\mathrm{WFAVG}_{f,t}} - 1\right)\right)$	$f \in \mathrm{FCAPNG}$ $a \in A$ $t \in T$	Allocation of new private capital by activity

S11	$QF_{f,a,t} = QF_{f,a,t-1}(1 - depr_{f,t-1}) + DKA_{f,t-1}$	$f \in FCAPNG$ $a \in A, t \in T$ $t \notin TMIN$	Accumulation of private capital by activity
S12	$WF_{f,t} = wfb_f$	$f \in FCAPNG$	Exogenous economy-wide wage term for private capital
S13	$QQ_{c,t} = \sum_{h \in H} QH_{c,h,t} + QG_{c,t} + QINV_{c,t} + \sum_{i \in INSD} qdstk_{c,i,t} + \sum_{a \in A} QINT_{c,a,t} + QT_{c,t}$	$c \in C$ $t \in T$	Commodity balance
S14	$\sum_{c \in C} pwe_{c,t} \cdot QE_{c,t} + \sum_{ac \in INSD \cup F} trnsfr_{ac,row,t} + SAVF_t = \sum_{c \in C} pwm_{c,t} \cdot QM_{c,t} + \frac{\sum_{i \in INSDNG} TRII_{row,i,t}}{EXR_t} + \sum_{f \in F} trnsfr_{row,f,t}$	$t \in T$	Current account of balance of payments
S15	$SAVF_t = \sum_{i \in INSD} nff_{i,t} + invf_t - drf_t + WALRAS_t$	$t \in T$	Capital account of balance of payments
S16	$\overline{CPI_t} = \sum_{c \in CGDP} \sum_{h \in H} PQD_{c,h,t} \cdot cwts_{c,h}$	$t \in T$	Consumer price index

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