PARENTAL CAREGIVING AND HOUSEHOLD POWER DYNAMICS

Ray Miller, Colorado State University
Neha Bairopiya, University of Southern California

CWE-GAM Working Paper Series: 20-05
Program on Gender Analysis in Economics (PGAE)
American University, Washington, DC
DOI: 10.17606/dr9x-vn55
MAY 2020

This working paper is part of the Care Work and the Economy (CWE-GAM) Project, based at American University’s Program on Gender Analysis in Economics (PGAE). The authors would like to acknowledge the financial support of the William and Flora Hewlett Foundation for this report. The findings, views, and conclusions or recommendations expressed in this material are strictly those of the authors and do not necessarily reflect those of the William and Flora Hewlett Foundation.

*Corresponding author email: ray.miller@colostate.edu
THE CARE WORK AND THE ECONOMY (CWE-GAM) PROJECT

The Care Work and the Economy (CWE-GAM) Project strives to reduce gender gaps in economic outcomes and enhance gender equality by illuminating and properly valuing the broader economic and social contributions of caregivers and integrating care in macroeconomic policymaking toolkits. We work to provide policymakers, scholars, researchers and advocacy groups with gender-aware data, empirical evidence, and analytical tools needed to promote creative, gender-sensitive macroeconomic and social policy solutions. In this era of demographic shifts and economic change, innovative policy solutions to chronic public underinvestment in care provisioning and infrastructures and the constraints that care work places on women's life and employment choices are needed more than ever. Sustainable development requires gender-sensitive policy tools that integrate emerging understandings of care work and its connection with labor supply, and economic and welfare outcomes.

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THE AUTHOR TEAM

RAY MILLER

Ray Miller is an Assistant Professor of Economics at Colorado State University (CSU). His research investigates the determinants and consequences of health disparities and social inequality. Research topics include the lasting impact of early health disparities, the welfare implications of health insurance, and the inequality of health and economic well-being among the elderly. Prior to joining the faculty at CSU, Miller spent three years as a Postdoctoral Research Associate at the Harvard Center for Population and Development Studies and Research Fellow in the Program on the Global Demography of Aging at Harvard.

NEHA BAIROLIYA

Dr. Bairoliya is an Assistant Professor in the Department of Finance and Business Economics at the Marshall School of Business at the University of Southern California. Bairoliya received her Ph.D. in Economics from the University of Minnesota in 2015. Her research interests lie in analyzing demographic transitions and understanding the macroeconomic and welfare effects of public policy programs globally. Dr. Bairoliya is particularly interested in examining healthcare and pension programs and understanding their role in mitigating health and socio-economic inequalities. Her research combines rigorous economic theory, the knowledge of public and social institutions, rich micro data and large-scale simulations to answer important questions at the intersection of public policy, population health and aging.
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1. INTRODUCTION

A growing concern in many countries is an aging population and an increase in the number of elderly in need of long-term care (de Meijer et al., 2013). However, the economic and welfare implications of elderly care provision remain relatively understudied. One of the primary factors which complicates welfare analysis is that a majority of elderly care is provided informally by family members, with adult children often comprising the largest share of care providers (Norton, 2000; Bettio and Verashchagina, 2010). The pervasiveness of unpaid care due to cultural or family ties has even limited the development of long-term care insurance in economically advanced regions like Europe (Costa-Font, 2010). However, while children may provide an informal safety net, parental caregiving is a time intensive task and must be met by adjustments along leisure or work margins on the part of the care provider. Hence, in order to understand the full macroeconomic implications of growing elderly care needs and the appropriate policy response, it is imperative to understand how households cope with these caregiving needs.

Economic models of elderly care have focused almost exclusively on inter-generational bargaining between parents and children or bargaining among siblings (e.g. Pezzin and Schone, 1999; Engers and Stern, 2002; Byrne et al., 2009; Barczyk and Kredler, 2017). However, little attention has been paid to the influence of care demands on the power dynamics between partners within a household (e.g. husband and wife). This is despite evidence that caregiving falls disproportionately on women (Barusch and Spaid, 1989; Bettio and Verashchagina, 2010) and that some caregivers respond to increased parental care needs by reducing work hours, taking more flexible jobs, or by quitting paid work entirely (Bauer and Sousa-Poza, 2015). Moreover, caregiving may have spillover effects on the caregiver’s spouse or partner. For example, a spouse may work longer hours or reduce their spending to cope with fewer hours of paid work by the caregiver. So, it is unclear to what extent partners reallocate their time and resources and share the welfare burden of parental care needs when they arise.

In this study, we develop a theoretical model to explore how unpaid parental caregiving can affect the allocation of time and resources across partners under different household power structures. In the standard “unitary model” pioneered by Becker (1981), a household behaves as if it were a single unit. In contrast, the pooling of resources across the household implied by the unitary model has been consistently and repeatedly rejected empirically (e.g. Lundberg et al., 1997; Fortin and Lacroix, 1997). We build a simple collective bargaining model of intra-household time and resource allocation in the spirit of Chiappori (1992). This approach views partners as individuals with conflicting preferences but who operate as a cooperative decision making unit.
As parental caregiving disproportionately falls on daughters (e.g. Bond et al., 1999; Bettio and Verashchagina, 2010; Haberkern et al., 2015), we consider a benchmark collective model in which a female’s parent may be in need of time intensive care. While the level of care needed is exogenous to the household, the provision of care is determined endogenously as a bargained outcome between the couple. This allows for some care needs to remain unmet, which is consistent with empirical findings in a variety of contexts (e.g. Herr et al. 2013; Bien et al. 2013). We show in the model how power dynamics and labor markets impact the time and resource allocation of caregiving partners and the unmet care needs of care recipients. We then use our theory with cross-country data from Europe to illustrate concepts and examine welfare with some numerical exercises.

We compare theoretical and numerical results under three alternate modeling assumptions. (1) **Exogenous bargaining power** for each partner and fully flexible labor markets. We use this as our baseline to examine how unpaid caregiving changes the allocation of time and consumption within a household under a given bargaining power structure. (2) **Endogenous bargaining power** dependent on the decisions of the household. Specifically, we assume bargaining position is tied to relative earnings.¹ A decline in labor earnings in response to unpaid caregiving can potentially weaken the caregiver’s bargaining position and affect the distribution of resources within the couple. Thus care provision may have additional implications for gender inequality if we take into account changing power dynamics within the household. (3) **Fixed labor supply** due to labor market rigidities. In the presence of labor market rigidities, any adjustments on the labor supply margin can be costly. Under such circumstances, caregiving needs must be met by adjustments solely along the leisure or home production margins.

We calibrate our theoretical model using cross-country data from the Survey of Health, Ageing, and Retirement in Europe (SHARE). In our quantitative exercise with exogenous bargaining power, we find that for a duel-earning couple in our benchmark country (France), 20 hours a week of care needs from the female’s parent results in a 26% welfare decline for the female and 15% for the male. In other words, the welfare burden of unpaid caregiving to the male is about 57% that of their female partner—a skewed but shared burden. The higher welfare cost to the female stems from two sources; (1) relatively fewer hours of leisure due to her provision of unpaid care; (2) the utility cost of leaving her parent with some level of unmet care needs. Comparing across countries, in those with the highest level of calibrated female bargaining power (e.g. Switzerland), the welfare burden can even fall more heavily on the male than the female, despite the female providing all the unpaid care and the care recipient being the female’s parent. This is a result of higher average labor market productivity of men driving an increase in male labor supply and a decrease in female labor supply. Moreover, countries with higher female bargaining power have lower levels of unmet care needs, suggesting power dynamics could have important implications for care recipients as well.

¹ In an extension, we also consider a threat point model in which bargaining power is tied to each partner’s best outside option.
In contrast to the shared burden under exogenous bargaining, the welfare cost of caregiving can become highly skewed against the caregiver when there is endogenous bargaining power. In our numerical example, the relative welfare cost for the French male falls from 57% to only 2% of that of the female. Moreover, unmet care needs increase more than 80%. More generally, countries with lower baseline female bargaining weights see the largest shift of the welfare burden towards the female caregiver and maintain the highest levels of unmet care needs. There are two related reasons for the shift in welfare burden. (1) Declines in female labor supply reduce the bargaining power of the female. (2) The female labor supply response is smaller due to the negative influence on bargaining power.

Lastly, removing labor market flexibility also results in significant welfare differentials within a household. For instance, when both men and women are unable to adjust work hours, the relative welfare cost to the male in France is 9% of that of the female. However, for a more equal country like Sweden, the relative welfare cost to the male is 25%. Moreover, only in the most equal countries in our sample do the partners split caregiving responsibilities. In most countries, the female continues to provide the entire amount of unpaid care, even when adjustments on the formal labor supply margin are eliminated. These are also the only countries where men increase home production when women provide unpaid care. Introducing labor market rigidities also yields the highest level of unmet care needs across models. For example, unmet care needs more than double compared to the benchmark case with flexible labor markets in France. Overall, our theoretical and numerical results show that ignoring bargaining power differentials can misrepresent the welfare effect of unpaid caregiving by not taking into account the uneven distributional consequences. In our model, a decrease in bargaining power increases an adult child’s share of the welfare burden and the unmet care needs of their parent. If bargaining power is endogenously determined by relative earnings, the welfare cost of caregiving can fall disproportionately on a single partner, resulting in a “triple burden” of market work, home production, and caregiving. Under this scenario, government policies subsidizing long-term care could decrease the welfare gap within a household by providing financial relief and improving the bargaining position of the caregiver. This could further result in reduced levels of unmet care needs and improved welfare of elderly care recipients. In general, labor market rigidities also exacerbate the total welfare cost of unpaid caregiving to the household as well as the unequal distribution of the burden. This implies policies that promote flexibility in number of working hours, such as caregiver leave or part-time options, could provide substantial relief, particularly to high intensity caregivers.
2. MODEL

Consider a household consisting of two working adults. For expositional convenience, we refer to household partners as female and male. Each member has their own utility function designating preferences over own consumption of market goods \( c_i \), domestically produced goods \( d_i \), and leisure \( l_i \). Utility is separable and given by:

\[
u(c_i, d_i, l_i) = \log c_i + \kappa \log d_i + \phi(l_i)\]

where \( \phi' > 0 \) and \( \phi'' < 0 \). Each member is endowed with a unit of time that is split between work in the formal labor market \( e_i \), hours devoted to domestic home production \( h_i \), and leisure \( l_i \):

\[e_i + h_i + l_i = 1.\] (1)

The household budget constraint is given by:

\[c_m + c_f = e_m + \gamma e_f\] (2)

where \( \gamma \) denotes the potential earnings differential between household members. Partners combine home production hours to produce domestic goods using a constant elasticity of substitution technology:

\[d_m + d_f = (\alpha h_m^\eta + (1 - \alpha) h_f^\eta)^{1/\eta}\] (3)

with \( \alpha \in (0,1) \) and \( \eta < 1 \).

Following the collective bargaining approach of Chiappori (1992), Pareto-efficient allocations of time and resources are derived by maximizing the weighted sum of partner utilities. Specifically, the partners maximize the household welfare function:

\[(1 - \theta)u(c_m, d_m, l_m) + \theta u(c_f, d_f, l_f)\]

subject to constraints (1)-(3). Here, \( \theta \in [0,1] \) measures the relative bargaining power of the female in the household. Note that with equal bargaining weights (a parameter value \( \theta = 0.5 \)), the collective bargaining allocation reduces to that of the standard unitary model.

2.1 EXOGENOUS BARGAINING POWER

First consider the case where bargaining power \( \theta \) is exogenous to the household. Bargaining power is likely influenced by cultural norms, prevailing female earnings potential, local institutions, and a variety of other external factors. In this case, the optimal
household consumption allocations follow a “sharing rule” as is typical in this type of collective bargaining model and stated in the following proposition:

**Proposition 1.** Each partner consumes a fraction of household income and domestic production equal to their bargaining weight:

\[
c_m = (1 - \theta)(e_m + \gamma e_f), \quad c_f = \theta(e_m + \gamma e_f)
\]

\[
d_m = (1 - \theta)(\alpha h_m + (1 - \alpha)h_f)^\frac{1}{\eta}, \quad d_f = \theta(\alpha h_m + (1 - \alpha)h_f)^\frac{1}{\eta}
\]

*Proof.* See appendix.

Turning next to female labor supply, the relevant first-order condition is given by:

\[
\frac{\gamma}{e_m + \gamma e_f} = \theta \phi'(1 - e_f - h_f).
\]

The household equates the marginal benefit of female labor to the weighted marginal cost. As an illustrative example, Figure 1 provides a graphical representation of equation (6), holding home production hours and male labor supply constant. We will refer to this curve as the labor supply curve. The curve is downward sloping, reflecting the fact that lower bargaining power increases female labor supply. As is clear from the figure, for a given set of parameters, a bargaining weight \( \theta < 0.5 \) results in higher female labor supply than the corresponding unitary model. Intuitively, as female bargaining power rises, she is afforded more leisure by lowering formal labor supply and time devoted to home production. The inverse is true for the male partner—increases in female bargaining weight \( \theta \) increases male labor supply. These results are more generally stated in the following proposition:

**Proposition 2.** Comparative statics for labor supply response to changes in exogenous bargaining power are given by:

\[
\frac{\partial e_f}{\partial \theta} < 0, \quad \frac{\partial e_m}{\partial \theta} > 0.
\]

*Proof.* See appendix.

---

2 One could also envision an alternate case where female labor supply increases with bargaining power if the male has a strong preference for the female to limit market work. This may be the case in more traditional or conservative societies. We focus on the standard case with downward sloping female labor supply curve. In our numerical exercise, we focus on European couples where both partners have strong labor force attachment, where the standard curve is likely to hold.
While bargaining power has clear implications for relative labor supplied across partners, this is not the case for hours devoted to domestic production. Combining household optimality conditions yields the following rule for relative allocation of home production hours across partners:

$$h_m = \left(\frac{\gamma a}{1 - a}\right)^{\frac{1}{1 - \eta}} h_f.$$  \hspace{1cm} (7)

This implies male home production hours will be lower than female whenever $\gamma < \frac{1 - a}{a}$. Intuitively, when female market returns $\gamma$ are small relative to her home production share $a$, females will spend more time at home relative to the male partner. Note also that relative home production does not depend on bargaining power weight $\theta$, but only on the relative productivity of home versus market work.

Finally, we can summarize the relative burden of home and market work across partners through the following leisure condition:

$$\phi'(l_m) = \frac{\theta}{(1 - \theta)\gamma} \phi'(l_f).$$  \hspace{1cm} (8)

This shows that the male partner will enjoy more leisure than the female whenever $\frac{\theta}{1 - \theta} < \gamma$. Conversely, if the female market return $\gamma$ is low relative to her bargaining weight $\theta$, she will enjoy more leisure than her partner.
2.1.1 PARENTAL CAREGIVING

Next consider the household equilibrium response to the realization of parental care needs. Clearly there are plausible arguments for treating the provision of unpaid care as a complex endogenous decision involving cultural norms, bargaining between caregiver and care recipient, and/or bargaining between potential caregivers (e.g. siblings). However, as our focus is on time allocation decisions within households of actual caregivers, we abstract from such consideration and simply treat total care needs as exogenous to the household $z \in (0,1)$. Unpaid care may be provided by the male ($z_m$) or female ($z_f$) partner. While the level of care need is exogenous, we allow for the time allocated to unpaid care to fall short of the full amount needed. Consistent with empirical evidence, this allows for the case in which care needs are only partially met. Let unmet parental care needs be given by $z_{gap} = z - z_m - z_f$.

As parental care provision disproportionately falls on daughters, we assume in our benchmark model it is the female’s parent in need of care. Specifically, we introduce the following modified household welfare function:

$$(1 - \theta)u(c_m, d_m, l_m) + \theta[u(c_f, d_f, l_f) - \omega(z - z_m - z_f)]$$

where $\omega' > 0$ and $\omega'' > 0$. The function $\omega(.)$ provides the female’s disutility from her parent’s unmet care needs, which is weighted by her relative bargaining power. With unpaid care, the modified time constraint for each partner and feasibility requirements on caregiving are given by:

$$e_i + h_i + l_i + z_i = 1$$

$$z_f + z_m \leq z, \quad z_f, z_m \geq 0.$$

Note that we assume hours spent in unpaid care are perfect substitutes across partners. In this case, it is optimal for the partner with lower market return to provide all unpaid care:

**Proposition 3.** If $e_f > 0$ and $\gamma < 1$, the female partner will provide all the unpaid care in the household: $z_f \geq 0$ and $z_m = 0$. If $e_m > 0$ and $\gamma > 1$, the male partner will provide all the unpaid care in the household: $z_f = 0$ and $z_m \geq 0$.

**Proof.** See appendix.

If the female earns less for market work than her partner, it is optimal for her to specialize in parental care. Moreover, this specialization comes at the cost of both forgone market work and home production:

**Proposition 4.** If $\gamma < 1$, interior comparative statics for labor supply and home production response to (exogenous) changes in unpaid caregiving are given by:
\[
\frac{\partial e_f}{\partial z_f} < 0, \quad \frac{\partial h_f}{\partial z_f} < 0, \quad \frac{\partial e_m}{\partial z_f} > 0, \quad \frac{\partial h_m}{\partial z_f} < 0.
\]

**Proof.** See appendix.

In contrast to the female, the male partner increases market work in response to increasing unpaid care by the female. However, the male lowers his home production in the same fashion as the female. This is due to the assumption that home inputs are less then perfect substitutes across partners. Therefore, lower female home production decreases the marginal return to male home production. Thus the male substitutes some hours out of home production and into market labor. Moreover, condition (7) continues to hold in the presence of increasing unpaid care. So not only do male and female home production move in the same direction, they remain in the same proportions.

Lastly, we can examine the relationship between care provision, unmet care needs, and bargaining power through the interior first-order condition for female unpaid care:

\[
\phi'(1 - e_f - h_f - z_f) = \omega'(z - z_f).
\]

Recall female labor supply \(e_f\) is decreasing in bargaining power \(\theta\) (Proposition 2) while the relationship between home production \(h_f\) and \(\theta\) is ambiguous. Consider the case in which the female partner weakly lowers her home production (or does not increase it too much) with increasing bargaining power. Then an increase in female bargaining power will also increase the provision of care \(z_f\) given that \(\phi'(\cdot)\) is increasing in \(e_f\) and \(z_f\) and \(\omega'(\cdot)\) is decreasing in \(z_f\). In other words, an increase in exogenous female bargaining power increases the provision of unpaid care and consequently lowers the equilibrium level of unmet care needs.

### 2.2. ENDOGENOUS BARGAINING POWER

It is plausible that bargaining power may not only depend on prevailing external factors but could also endogenously evolve depending on the decisions of the household. In particular, theoretical and empirical research suggests that one’s relative earnings within the household may play an important role (Mencher, 1988; Blumberg and Coleman, 1989; Desai and Jain, 1994; Riley, 1997; Lundberg et al., 1997; Attanasio and Lechene, 2002; Bonke and Browning, 2009). In this case, we may think \(\theta\) is dependent on the distribution of income within the household (Basu, 2006).

In this spirit, we consider an endogenously determined bargaining power defined by \(\theta = \theta(E)\), where \(E = \frac{e_f}{e_m}\) is the female to male earnings ratios. We assume \(\theta'(E) \geq 0\) so that bargaining power is increasing in relative earnings. For a given value of \(\theta\), the household maximizes their previously defined welfare function. However, this may in turn cause \(\theta\) to
change, resulting in further desired adjustments. Following the idea of Basu (1999; 2006), we consider the stationary point of this process as the equilibrium of interest. Denoting the solution to the household maximization problem given $\theta$ as $q(\theta) = \{c_m, c_f, d_m, d_f, e_m, e_f, h_m, h_f\}$, the equilibrium of this adjustment process can be defined as follows:

**Definition.** A household equilibrium with endogenous bargaining power is a vector of outcomes $q^*$ and a power index $\theta^*$, such that $\theta^* = \theta(E^*)$, and $q^* = q(\theta^*)$, where $E^*$ is the earnings ratio arising from outcome vector $q^*$.

Following this definition and using the household’s first-order condition without parental care needs (6), a female labor supply allocation $e^*_f$ is part of an equilibrium if

$$\frac{\gamma}{e^*_m + \gamma e^*_f} = \theta(E^*) \phi'(1 - e^*_f - h^*_f).$$

As an illustration, Figure 2 plots the labor supply curve defined by (6) along with a hypothetical endogenous bargaining power curve $\theta(E)$—holding male labor supply constant. The bargaining power curve is upward sloping in female labor supply as more market work increases the female’s relative earnings. The equilibrium described by (9) is given by the intersection of the two curves.

How does incorporating endogenous bargaining power change the predictions of the model when parental care is introduced? Incorporating care need $z$ into the household welfare function yields a modified first-order condition for female labor supply:

$$\frac{\gamma}{e^*_m + \gamma e^*_f} = \theta \phi'(1 - e^*_f - h^*_f - z^*_f).$$

Figure 2: Endogenous bargaining power and female labor supply
Figure 3 plots the female labor supply curve with and without unpaid caregiving. Introducing unpaid care shifts the labor supply curve to the left. Consider starting in the equilibrium of the non-caregiving household \((e_f, \theta)\). In the case of exogenous bargaining power, the caregiving requirement results in new equilibrium \((e'_f, \theta)\). In contrast, the new endogenous bargaining power equilibrium occurs at point \((e''_f, \theta'')\), where the new labor supply curve intersects the power curve. Given the upward slope of the power curve, it is clear that \(\theta > \theta''\) and \(e'_f < e''_f\). The female labor supply response to unpaid caregiving is weaker with endogenous bargaining power. The magnitude of this difference will depend on the shape and slope of the power curve.

![Bargaining power curve](image)

**Figure 3: Female labor supply response to unpaid care shock**

A weak labor supply response is generally consistent with empirical evidence that suggests there is a negative but often relatively small effect of caregiving on hours worked (Bauer and Sousa-Poza, 2015). Several mechanisms have been proposed as potential explanations for the small labor response. These include a countervailing wealth effect due to either increased expenses associated with providing unpaid care (e.g. food, medicine, etc.) or wage declines due to less work flexibility (Twigg and Atkin, 1994; Heitmueller and Inglis, 2007). A “respite effect” has also been proposed where unpaid caregivers prefer work in order to get away from their caregiving responsibilities (Twigg and Atkin, 1994). Here we are considering a novel complementary mechanism that operates through the distribution of bargaining power in the household.

This simple framework makes sharp predictions about the difference in labor supply response to parental caregiving given exogenous versus endogenous bargaining power. In contrast, the difference in labor supply response between the unitary and exogenous bargaining power models is theoretically ambiguous. More generally, whether the labor supply response to caregiving needs increases or decreases with an exogenously given \(\theta\)
will depend on functional forms and parameter values. In our calibrated numerical example we show that differences can feasibly be quite large. The next section also explores the possibility that labor markets are rigid, and thus some caregivers (and their partners) are simply unable to adjust along the work margin.

2.3 FIXED LABOUR SUPPLY

Empirical evidence suggests unpaid caregiving lowers labor supply on both the extensive and intensive margins. However, the effects of caregiving on employment and hours worked are often found to be relatively small overall, particularly for low intensity care (Bauer and Sousa-Poza, 2015). Our model has so far assumed fully divisible labor and frictionless labor markets. This allows partners to freely allocate hours between market work, home production, and leisure. If females could only operate on the extensive margin, they may be unable to optimally lower their labor supply in the presence of parental care needs. Likewise, the desired increase in male labor hours may be infeasible if additional overtime or shifts are unavailable. Moreover, if there are re-employment costs and unpaid caregiving is unlikely to persist for an extended period of time, females may be unwilling to lower labor supply to the same extent as the static frictionless model predicts. This may be why becoming a caregiver reduces labor force participation but leaving the caregiver role has no effect on the probability of re-entry into the labor market (Spiess and Schneider, 2003; Wakabayashi and Donato, 2005; Van Houtven et al., 2013). In this section, we examine the theoretical implications of assuming fixed labor supply in our model.

An important implication of fixed labor supply is that unpaid caregiving no longer always falls entirely to the female partner. Given the optimal allocation of home production with fixed labor supply, the relevant first-order condition for unpaid care allocation is given by:

$$
\theta \phi'(1 - \bar{e}_f - h_f - z_f) = (1 - \theta)\phi'(1 - \bar{e}_m - h_m - z_m).
$$

Partners divide caregiving time to equalize their marginal cost of lost leisure, weighted by relative bargaining power. With large enough care need $z$, it is possible that this interior solution is reached, and partners share caregiving responsibilities. This contrasts the model with flexible labor supply, where the female always provides all the unpaid care (under the conditions of Proposition 3).

At an interior solution where both partners provide some care, home production allocation is determined by the modified condition

$$
h_m = \left[\frac{\alpha}{1 - \alpha}\right]^{\frac{1}{1 - \eta}} h_f.
$$

Comparing to the analogous condition with endogenous labor (7), the optimal distribution of home production is still independent of bargaining weights. However, this allocation no
longer depends on market productivity $\gamma$, as the labor supply margin is eliminated. Instead, the distribution is entirely determined by home production technology parameters.

If parental care needs are small enough, caregiving will continue to fall on a single partner and equation (11) need no longer hold. In this case, the home production allocation condition becomes

$$h_m = \left( \frac{\alpha \theta \phi' (l_f)}{(1 - \alpha)(1 - \theta) \phi' (l_m)} \right)^{\frac{1}{1-\eta}} h_f.$$ (13)

Relative home production now depends on bargaining power with male hours increasing in $\theta$. Intuitively, as home hours are imperfect substitutes, it is optimal to use bargaining power to adjust time along other more substitutable dimensions (i.e. work or caregiving). However, with fixed labor supply and male care provision already at zero, the only margin left to utilize one’s bargaining weight is home production.

3. NUMERICAL EXAMPLE

In order to illustrate the mechanisms of our theoretical model, we provide a simple numerical exercise where we calibrate parameters and simulate the influence of parental care needs on consumption and time allocations within a household. Specifically, we compare a baseline of no care needs ($z = 0$) to an equilibrium with 20 hours of care needs per week ($z = 0.17$).³ We compare results under our three alternate modeling assumptions; (1) exogenous bargaining power and fully flexible labor markets; (2) endogenous bargaining power and fully flexible labor market; (3) fixed labor supply due to labor market rigidities. As we detail below, the baseline with no care needs will be identical across the three modeling assumptions. However, predictions will differ once parental care needs are introduced. We begin with a more detailed analysis of a duel-earning household from a single country (France) before conducting a more general cross-country comparison. In later extensions, we also consider alternate household compositions.

3.1 FRANCE

3.1.1. CALIBRATION

In order to numerically calibrate our model, we use data primarily from the Survey of Health, Ageing, and Retirement in Europe (SHARE), a longitudinal study of individuals aged

³ We convert all weekly hours to our model by assuming a time endowment of 16 hours a day, 365 days a year, and 50 weeks of work/care. For example, 20 hours a week yields $z = (20 \times 50)/(365 \times 16) = 0.17$. 
50 or older and their partners covering 27 European countries and Israel. SHARE data contains information on socioeconomic-status and social and family networks, including labor market outcomes and time spent in unpaid caregiving. There are currently six waves of SHARE available, collected biennially between 2004 and 2017. We use SHARE data on gender, age, country of residence, weekly hours worked, earnings, and caregiving.

As our framework incorporates only limited heterogeneity, we restrict the SHARE sample to as homogeneous a population as feasible for our baseline exercise. After pooling across all survey waves, we retain observations for individuals aged 40-59 who live with a partner. This age range captures a substantial share of parental caregivers while limiting cases of care provision to one’s partner. Limiting the sample to those under 60 also lessens concerns over simultaneous retirement and caregiving decisions. We define an unpaid caregiver as anyone that reported giving personal care or practical household help “about daily” to someone in the previous twelve months.4 We calibrate parameters using wage and hours worked SHARE data from couples in which both partners are non-caregivers working at least 20 hours a week and neither partner requires care themselves. In other words, we calibrate the model to a household with no caregiving and in which both partners are substantially attached to the labor market. In later extensions, we also examine single females and households with a single market earner.

Table 1: Calibration for France

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Targets / Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Earnings</td>
<td>γ</td>
<td>0.76</td>
<td>Gender wage ratio 0.76</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>ε</td>
<td>1.00</td>
<td>Frisch elasticity 05.-2.0</td>
</tr>
<tr>
<td>Utility weight on l</td>
<td>ν</td>
<td>6.10</td>
<td>Male hours worked 42.5</td>
</tr>
<tr>
<td>Female bargaining power</td>
<td>θ*</td>
<td>0.42</td>
<td>Female hours worked 37.1</td>
</tr>
<tr>
<td>Home hours substitutability</td>
<td>η</td>
<td>0.49</td>
<td>Male home hours 11.9</td>
</tr>
<tr>
<td>Utility weight on d</td>
<td>κ</td>
<td>0.39</td>
<td>Female home hours 20.5</td>
</tr>
<tr>
<td>Home hours male share</td>
<td>α</td>
<td>0.50</td>
<td>---</td>
</tr>
<tr>
<td>Endogenous power curve</td>
<td>b</td>
<td>0.25</td>
<td>Earnings ratio 0.66</td>
</tr>
<tr>
<td>Disutility of unmet care</td>
<td>δ</td>
<td>3.05</td>
<td>Unmet care needs 25%</td>
</tr>
</tbody>
</table>

Table 1: summarizes the baseline calibration for France. We first set relative female earnings potential γ = 0.76 to equal the aggregate wage ratio of females to males in France from our SHARE sample.5 We then define standard preferences over leisure given by:

\[
\phi(l) = -\frac{νε}{1 + ε} (1 - l)^{1+ε},
\]

where ε is a constant Frisch elasticity of labor supply (the elasticity of labor supply with respect to wage, holding the marginal utility of consumption fixed). Empirical studies of

4 This includes reportedly helping others outside or inside the household.
5 Hourly wage calculated as reported annual earnings divided by reported weekly hours worked times 52.
the Frisch elasticity vary considerably, with estimates ranging from 0.5 to nearly 2 (Hall, 2009; Chetty, 2012). We choose a value of $\epsilon = 1$. Given this form, equation (8) can be written:

$$\theta = \frac{\gamma(e_m + h_m)}{e_f + h_f + \gamma(e_m + h_m)}.$$  

(14)

Plugging this equation for $\theta$ into condition (6) yields the following expression for the utility weight on leisure:

$$\nu = \frac{e_f + h_f + \gamma(e_m + h_m)}{(e_f + h_f)(e_m + \gamma e_f)(e_m + h_m)}.$$  

(15)

For home production technology, we assume $\alpha = 0.5$, implying partners are equally productive in the production of home goods. Condition (7) may then be written:

$$\eta = 1 - \log \frac{h_m}{h_f}.$$  

(16)

Finally, combining household first-order conditions for labor supply and home production yields the following equation for utility weight on home consumption:

$$\kappa = \frac{(1 + \gamma \eta^{-1}) h_m}{e_m + \gamma e_f}.$$  

(17)

Equations (14)-(17) express four parameters $\{\nu, \theta, \eta, \kappa\}$ as functions of four equilibrium moments $\{e_f, e_m, h_f, h_m\}$. We obtain our calibrated parameter values by estimating these moments from the data and plugging them into the equations. We estimate an average male (female) labor supply of 42.5 (37.1) hours per week for France from our SHARE sample. As SHARE lacks detailed data on time use outside of the formal labor market, we estimate home production hours with data on hours spent on “unpaid domestic work” from the Multinational Time Use Study (MTUS), a harmonized collection of time diary data. We use data collected in 2009-10 for France and limit the sample to those aged 40-59 in which the individual and their partner works full-time to best approximate our sample used from SHARE. We estimate average male (female) home production of 11.9 (20.5) hours a week from the MTUS for France.

When moving to endogenous bargaining power, we additionally need to parameterize the bargaining power function $\theta(E)$. First note that equations (14)-(17) must still hold in equilibrium when bargaining power is endogenous so the calibrated parameter values and baseline equilibrium are identical to the case of exogenous bargaining power. However, when introducing parental care, the functional form of $\theta(E)$ plays a crucial role in the model’s predictions when bargaining power is endogenous. As is clear from Figure 2, any decrease in female labor supply due to parental care can be rationalized in the model by selecting the appropriate change in bargaining power. Moreover, unlike preference
parameters, there is no standard approach or directly applicable research to help credibly pin down the bargaining power function. As such, we choose a simple linear functional form:

$$\theta(E) = (0.5 - b)E + b.$$  \hspace{1cm} (18)

This form implies $\theta(1) = 0.5$, or that equal earnings yields equal bargaining power. Denoting the baseline equilibrium earnings ratio $E^*$, it must that $\theta^*(E^*) = (0.5 - b)E^* + b$. As baseline $\theta^*$ is pinned down by equation (14), we estimate $E^*$ from our SHARE sample and obtain $b = \frac{\theta^* - 0.5E^*}{1 - E^*} = 0.25$.

Given the potential sensitivity of numerical results to the form of $\theta(E)$, we compare predictions from (18) with those from which bargaining power is exogenously held fixed at the baseline level: $\theta(E) = \theta^*$. Exogenous bargaining power can be viewed as the limiting case of the endogenous model, where the bargaining power curve becomes horizontal. We also consider the case where male and female labor supply are fixed at their baseline values. Note that with fixed labor supply the earnings ratio $E$ is also fixed, so there is no distinction between exogenous and endogenous bargaining power.

This leaves only the disutility of unmet care needs to be calibrated. We assume preferences over unmet care needs are given by:

$$\omega(z - z_m - z_f) = \delta \left( \exp \{ z - z_m - z_f \} - 1 \right),$$

where $\delta$ is the disutility weight on any parental care needs that are not provided by the household. Empirical estimates of unmet care needs can vary significantly depending on definition and data source. The estimated share of elderly with unmet care in France has ranged from 23%-51% of those with a need for care (Gannon and Davin, 2010; Herr et al., 2013). Shortfall in hours of care below the optimal is even more difficult to empirically pin down. For simplicity, we calibrate a care gap of 25%, or five hours, for our benchmark exogenous bargaining power case in France.\(^6\) This results in $\delta = 3.05$, which is held fixed across all modeling assumptions (e.g. endogenous bargaining power or fixed labor supply). This seems a plausible starting point and, more importantly, allows for comparison of predicted unmet care needs across differing countries and model assumptions.

### 3.1.2. WELFARE

In addition to examining differences in consumption and time allocation patterns across partners, we are also interested in the distribution of welfare costs. We use a consumption-equivalent variation (CEV) measure to quantify the difference in welfare effects of unpaid caregiving across partners. Our welfare measure is akin to asking by what percentage market consumption has to be decreased (holding leisure and home

---

\(^6\) This implies 15 hours of care a week which qualifies as "high intensity" caregiving as often defined using a threshold of 10-20 hours a week (Heitmueller and Inglis, 2007; Lilly et al., 2010; King and Pickard, 2013).
consumption constant) to make an individual indifferent to the household providing unpaid care. Formally, male welfare $\lambda_m$ is defined by the condition:

$$u((1 + \lambda_m)c_m, d_m, l_m) = u(c_m^z, d_m^z, l_m^z),$$

where $z$ superscripts denote equilibrium outcomes associated with household level of care need $z$. Under the assumed log preferences, the welfare condition may be explicitly written:

$$\lambda_m = \exp(u(c_m^z, d_m^z, l_m^z) - u(c_m, d_m, l_m)) - 1.$$

For example, a $\lambda_m = -0.1$ implies the male partner would be indifferent between giving up 10% of his baseline market consumption or the household equilibrium outcomes associated with care need $z$.

Female welfare $\lambda_f$ is given by the modified condition:

$$\lambda_f = \exp(u(c_f^z, d_f^z, l_f^z) - \omega(z - z_m - z_f) - u(c_f, d_f, l_f)) - 1.$$

The only difference with male welfare is that the female directly values caregiving and is increasingly hurt by higher levels of unmet care needs. To be clear, while we will sometime refer to our welfare results as the cost of caregiving, there is also an included welfare cost for the female due to any care needs of her parent that are left unmet.

### 3.1.3. RESULTS

We next compare equilibrium allocations with and without parental care needs. Specifically, we compare the baseline of no care needs ($z = 0$) to an equilibrium with 20 hours of care needs per week ($z = 0.17$). A summary of equilibrium results is provided in Table 2. Let us first focus on results under exogenous bargaining power. Recall that by Proposition 3, it is optimal for the female to provide all 15 hours of unpaid care. As a result, female labor supply is 10.4 hours (28.1%) lower and home production 1.2 hours (5.6%) lower with unpaid caregiving. The remaining 3.4 hours devoted to care comes at the expense of female leisure time. While the male partner in the caregiving household does not provide any care, they increase their labor supply 9.2% (3.9 hours per week) and reduce home production the same proportion as the female (in accordance with Proposition 4 and condition (7)).

Net declines in market and home production lead to lower consumption levels with unpaid care. Moreover, as $\theta$ is held fixed, the sharing rule does not change and the percentage decline in market and home consumption levels are exactly equal for both partners (5.6%). Unmet care combined with substantially less leisure time and lower consumption levels for the caregiving female results in a welfare measure $\lambda_f = -0.26$. The non-caregiving female would give up to 26% of her market consumption to avoid the equilibrium

---

7 This is roughly consistent with data from our SHARE sample where more than 80% of caregiving households reported only one daily caregiver.
outcomes of the caregiving female. In contrast, the male partner would only give up 15% of his market consumption to avoid the unpaid care equilibrium. In other words, the welfare burden of unpaid caregiving to the male is only 57% that of their female partner.

Turning to the case of endogenous bargaining power, the welfare burden is even less equally distributed across partners. With the threat of lost bargaining weight, the female caregiver reduces market labor only 4.5 hours compared to the no care baseline. The male partner increases market work only 0.6 hours—less than a sixth of the increase with exogenous bargaining power. As a result, the leisure time cost associated with caregiving is much less evenly borne across partners.

Table 2: Equilibrium with and without parental care (France)

<table>
<thead>
<tr>
<th></th>
<th>Exogenous $\theta$</th>
<th>Endogenous $\theta$</th>
<th>Fixed labor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Care</td>
<td>% Change</td>
<td>Care</td>
</tr>
<tr>
<td>$z_f$</td>
<td>---</td>
<td>15.0</td>
<td>---</td>
</tr>
<tr>
<td>$z_{gap}$</td>
<td>---</td>
<td>5.0</td>
<td>---</td>
</tr>
<tr>
<td>$e_f$</td>
<td>37.1</td>
<td>-28.1</td>
<td>32.6</td>
</tr>
<tr>
<td>$e_m$</td>
<td>42.5</td>
<td>9.2</td>
<td>43.1</td>
</tr>
<tr>
<td>$h_f$</td>
<td>20.5</td>
<td>-5.6</td>
<td>19.7</td>
</tr>
<tr>
<td>$h_m$</td>
<td>11.9</td>
<td>-5.6</td>
<td>11.4</td>
</tr>
<tr>
<td>$l_f$</td>
<td>59.2</td>
<td>-5.8</td>
<td>53.6</td>
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<tr>
<td>$l_m$</td>
<td>62.4</td>
<td>-5.2</td>
<td>62.3</td>
</tr>
<tr>
<td>$c_f$</td>
<td>1.00</td>
<td>-5.6</td>
<td>0.91</td>
</tr>
<tr>
<td>$c_m$</td>
<td>1.40</td>
<td>-5.6</td>
<td>1.40</td>
</tr>
<tr>
<td>$d_f$</td>
<td>0.23</td>
<td>-5.6</td>
<td>0.21</td>
</tr>
<tr>
<td>$d_m$</td>
<td>0.32</td>
<td>-5.6</td>
<td>0.31</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.42</td>
<td>0.0</td>
<td>0.39</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>---</td>
<td>-0.26</td>
<td>---</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>---</td>
<td>-0.15</td>
<td>---</td>
</tr>
<tr>
<td>$\lambda_f/\lambda_m$</td>
<td>0.57</td>
<td>---</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Notes: \( z, z_f, e_f, e_m, h_f, h_m, l_f, l_m \) converted to weekly hours. Market and home consumption levels normalized: \( c_f = 1 \). Equilibrium with care needs of 20 hours per week. The baseline (no care) equilibrium is identical across modelling assumptions.

The lower female (and higher male) labor supply results in a lower earnings ratio and bargaining weight in the care providing household—\( \theta = 0.39 \) compared to 0.42 in the no care baseline. As the bargaining weight directly determines the household sharing rule, market and home consumption is significantly lower (8.9%) for the caregiving female, but only slightly lower (0.3%) for their male partner. The associated welfare cost for the female is 41% of baseline market consumption compared to 1% for the male—a ratio of 0.02. Moreover, as the female must consider the trade-off between lost bargaining power and providing care, unmet care needs are 9.1 hours compared to 5.0 hours with exogenous bargaining power. Thus there is also an implied shift in the welfare cost towards the care recipient, though explicitly quantifying this welfare cost is outside the scope of the current model.

The final columns in Table 2 show the equilibrium for France with 20 hours of care needs but holding male and female labor supply fixed at the baseline (no care) level. Note that with fixed labor supply, bargaining power remains constant regardless of whether it is exogenous or endogenously determined. In France, the non-negativity constraint on male caregiving binds, and the female provides all unpaid care. In order to allow for this care without changing labor supply, she reduces home production by 2.8 hours and leisure by 6.5 hours. In this case, the French female in the no care equilibrium is willing to give 39% of her market consumption to avoid the caregiving equilibrium. This welfare cost falls in between those estimated with endogenous and exogenous bargaining power and flexible labor supply.

In lieu of increasing labor supply, the French male increases home production in the presence of parental care needs. This results in a small 0.8% decline in leisure. He also suffers from lower consumption of the domestic good due to lower female home production hours. On net, the welfare cost of caregiving to the French male is 4% of baseline market consumption, or about 9% of the welfare loss of the female. Moreover, unmet care needs with fixed labor supply reaches 10.7 hours, the highest across the three models. This highlights the potential spillover value of flexible labor markets on care recipients.

### 3.2. CROSS-COUNTRY

We turn now to a cross-country comparison of duel-earning couples in European countries with available data in SHARE. The primary objective of this comparison is to
highlight how differences in bargaining power structures influence the distribution of welfare when parental care needs are introduced.\(^8\)

### 3.2.1 CALIBRATION

Parameters governing preferences and home production are assumed to be common across all countries and are left at the values calibrated for France.\(^9\) However, we allow two parameters to vary across countries in our numerical exercise. First, we change relative productivity \(\gamma_z\) to the aggregate wage ratio between females and males reported in our SHARE sample for dual-earning couples in each country. Second, we calibrate baseline bargaining power \(\theta^*_z\) to match the ratio of female to male hours worked for each country (Proposition 2 ensures identification of this moment).\(^10\) Similar to France, for \(\theta^*_z\) to be an equilibrium with endogenous bargaining power in country \(c\), it must be that 

\[
\theta^*_z(E^*_z) = (0.5 - b_c)E^*_z + b_c,
\]

where \(E^*_z\) is the baseline equilibrium earnings ratio in the country. This implies the power earnings function will differ across countries through parameter \(b_c\). Differences in \(b_c\) across countries reflects differences in institutions, culture, laws, and other factors that map relative earnings into bargaining power.

### 3.2.2. BASELINE FEATURES

The first four columns in Table 3 provide a comparison of average hours worked in the baseline model (no parental care needs) and the data for each country. Average female labor supply in our SHARE sample ranged from 30.0 hours a week in the Netherlands to 42.9 hours in Poland. There was less variation in male labor supply, ranging from 40.8 hours in the Netherlands to 48.3 in Israel. The gender gap in labor supply ranged from 2.5 hours a week in Estonia to 13.3 hours in Switzerland. Recall our baseline was calibrated to match relative labor supply between genders within each country. Overall, the baseline model also gives a reasonable approximation of average labor supply levels for males and females across countries (correlation coefficient of 0.82 for female labor supply and 0.12 for male).

The last five columns in Table 3 provide some additional features of the baseline calibration. Gender wage ratios \(\gamma\) (estimated directly from SHARE data) ranged from 0.71 in Germany to 0.91 in Sweden and Belgium. As expected, higher wage ratios are associated with higher female bargaining weights \(\theta\) (correlation coefficient of 0.9).

---

\(^8\) An alternate option would be to exogenously change parameters for France and examine results. However, we think a cross-country comparison more directly grounds the analysis in data and allows more intuitive interpretation of results.

\(^9\) Given that we use log preferences, scaling productivity across countries in market or home production does not change our results. Any increase in productivity only scales up consumption of that good while welfare results and allocations of time are unaffected.

\(^10\) We are unable to use equation (14) to directly pin down \(\theta^*_z\) for each country because we do not have hours of home production.
Females in a country with a large estimated gender wage gap generally have lower bargaining power than in countries with small wage gaps (e.g. $\theta = 0.40$ in Poland compared to $\theta = 0.49$ in Sweden). However, even countries with similar wage ratios can differ in calibrated bargaining weights based on observed labor supply gaps. Take the case of Switzerland and Italy. Both countries have the same gender wage ratio: $\gamma = 0.81$. However, the targeted labor supply ratio is 0.89 in Italy compared to 0.71 in Switzerland. In order to rationalize this difference within the structure of the current model, it must be that the baseline equilibrium bargaining weight is lower in Italy than in Switzerland. This lower $\theta$ results in more hours supplied by females in Italy and hence, rationalizes the observed higher labor supply ratio.

Table 3: Cross-county calibration

<table>
<thead>
<tr>
<th>Model fit</th>
<th>Baseline features</th>
<th>$e_f$</th>
<th>$e_m$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$e_f/e_m$</th>
<th>$h_f$</th>
<th>$l_f/l_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>37.1</td>
<td>37.1</td>
<td>42.5</td>
<td>42.5</td>
<td>0.76</td>
<td>0.42</td>
<td>0.87</td>
<td>1.72</td>
</tr>
<tr>
<td>Austria</td>
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<td>35.1</td>
<td>44.7</td>
<td>43.9</td>
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<td>0.43</td>
<td>0.80</td>
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</tr>
<tr>
<td>Germany</td>
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<td>35.9</td>
<td>43.4</td>
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<td>0.71</td>
<td>0.40</td>
<td>0.83</td>
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</tr>
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<td>37.3</td>
<td>43.2</td>
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<td>0.91</td>
<td>0.49</td>
<td>0.87</td>
<td>1.20</td>
</tr>
<tr>
<td>Netherlands</td>
<td>30.0</td>
<td>33.3</td>
<td>40.8</td>
<td>45.3</td>
<td>0.75</td>
<td>0.44</td>
<td>0.74</td>
<td>1.77</td>
</tr>
<tr>
<td>Spain</td>
<td>38.1</td>
<td>36.9</td>
<td>44.1</td>
<td>42.7</td>
<td>0.81</td>
<td>0.44</td>
<td>0.86</td>
<td>1.52</td>
</tr>
<tr>
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<td>42.9</td>
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<td>0.81</td>
<td>0.44</td>
<td>0.89</td>
<td>1.52</td>
</tr>
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<td>Denmark</td>
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<td>42.3</td>
<td>0.89</td>
<td>0.47</td>
<td>0.89</td>
<td>1.27</td>
</tr>
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<td>Greece</td>
<td>40.9</td>
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<td>46.9</td>
<td>42.5</td>
<td>0.76</td>
<td>0.42</td>
<td>0.87</td>
<td>1.70</td>
</tr>
<tr>
<td>Switzerland</td>
<td>33.5</td>
<td>33.0</td>
<td>46.8</td>
<td>46.1</td>
<td>0.81</td>
<td>0.48</td>
<td>0.71</td>
<td>1.50</td>
</tr>
<tr>
<td>Belgium</td>
<td>35.5</td>
<td>36.0</td>
<td>43.3</td>
<td>44.0</td>
<td>0.91</td>
<td>0.50</td>
<td>0.82</td>
<td>1.20</td>
</tr>
<tr>
<td>Israel</td>
<td>38.3</td>
<td>35.1</td>
<td>48.3</td>
<td>44.2</td>
<td>0.81</td>
<td>0.46</td>
<td>0.79</td>
<td>1.52</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>42.0</td>
<td>38.3</td>
<td>45.6</td>
<td>41.6</td>
<td>0.78</td>
<td>0.42</td>
<td>0.92</td>
<td>1.63</td>
</tr>
<tr>
<td>Poland</td>
<td>42.9</td>
<td>38.0</td>
<td>47.2</td>
<td>41.8</td>
<td>0.74</td>
<td>0.40</td>
<td>0.91</td>
<td>1.80</td>
</tr>
</tbody>
</table>
Notes: $e_f, e_m$ converted to weekly hours. Data estimates for SHARE sample aged 40-59, both partners working at least 20 hours a week, and neither partner providing unpaid care.

As an external comparison, Figure 4 plots our estimated bargaining weights against the United Nations Development Programme’s Gender Inequality Index (GII). The GII is a composite measure that incorporates gender inequality on dimensions related to reproductive health, political and educational empowerment, and labor market participation. Notably, the GII suggests the calibrated bargaining weight may be somewhat too high in Belgium and too low in Germany and Poland. However, overall bargaining power maps reasonably well to the GII (correlation coefficient of $-0.66$).

![Figure 4: Bargaining power and Gender Inequality Index](image)

Across all countries, our model predicts that the female partner supplies more hours to home production than the male—ranging from 20% more in Sweden to nearly double in Germany. In Switzerland, where female bargaining power is high but the wage ratio is about average, the female partner enjoys 12% higher leisure than the male. In contrast, low female bargaining power in Poland and Estonia results in a female leisure time equal to only 91% of her male partner.

3.2.3. RESULTS

Figure 5 plots the relative welfare costs of 20 hours of care needs against each country's baseline bargaining weight. Results are provided for exogenous and endogenous
bargaining power and fixed labor supply. In all cases, the welfare burden of unpaid care shifts towards the male partner as $\theta$ increases. Recall the relative welfare cost to the male in France was estimated at 57% of the female with exogenous bargaining power ($\lambda_m/\lambda_f = 0.57$). This estimate for France is included in panel (a) of the figure. As demonstrated in panel (a), at high enough (fixed) bargaining power, the welfare burden of unpaid care is shifted disproportionately onto the male partner (e.g. Switzerland). This occurs even though the female continues to provide the entire amount of unpaid care. The decline in male leisure time driven by an increase in male labor supply is larger than the decline in female leisure associated with unpaid caregiving. It is because the male partner earns a higher return in the labor market that it is optimal for him to shoulder a larger share of the welfare burden when bargaining power is fixed and relatively equal. However, as with France, when female bargaining power is relatively low, she will bear a larger share of the burden than the male partner.

Figure 5: Relative welfare cost of 20 hours of care needs ($\lambda_m/\lambda_f$)

With endogenous bargaining power the relative welfare cost to the male in France was estimated at 2% of the female ($\lambda_m/\lambda_f = 0.02$). As shown in panel (b) of Figure 5, in a few countries (e.g. Estonia), male welfare is slightly higher in the unpaid care equilibrium due to
increased bargaining weight (i.e. the relative welfare cost is negative). In contrast, the relative burden is about half as high for males in Sweden and Switzerland and fully shared between genders in Belgium.

Panel (c) of Figure 5 plots welfare costs holding male and female labor supply fixed at the baseline (no care) level. Consider Sweden as a complementary case to the French results previously discussed. In Sweden, female bargaining power is high enough that caregiving is divided between partners—10.6 hours provided by the female and 2.1 hours by the male. As the unpaid care equilibrium is an interior solution for Sweden, condition (12) holds and home production is equal across partners.\textsuperscript{11} Compared to France, the leisure cost of unpaid care is more evenly divided between partners as well—12.5% loss for the female and 3.8% for the male. These allocations lead to a shift in the welfare cost of care from the female towards the male, with a welfare ratio of 25% for Sweden (compared to 9% for France). The comparison between France and Sweden highlights the importance of bargaining power in driving the relative welfare burden across partners when labor supply is held fixed. Only in three countries with relatively high welfare ratios—Belgium, Sweden, and Denmark—are caregiving hours divided between partners. In all other countries, care continues to fall entirely on the female. Finally, note that when the labor supply margin of adjustment is eliminated, the welfare ratio is more condensed across countries—ranging from 8% in Poland to 34% in Belgium.

Turning to the provision of care, Figure 6 plots hours of unmet care needs for each country across modeling assumptions. In all cases, hours of unmet care needs generally falls with increases in female bargaining weight. For example, with exogenous bargaining power, there are no unmet needs in Belgium but almost seven hours in Poland (recall unmet care needs were calibrated to equal five hours in France in this case). Results are generally consistent with the estimates of Bien et al. (2013), who find more unmet needs among Southern-Eastern European countries compared to Northern-Western European countries. As with France, predicted unmet care needs are generally lowest when bargaining power is exogenous and labor markets are flexible. Unmet care needs are highest (and most condensed across countries) when labor supply is held fixed.

\textsuperscript{11} In our numerical exercise $\alpha = 0.5$, so condition (12) simplifies to $h_m = h_f$. 

4. EXTENSIONS

We next consider several extensions to the model exploring the implications of household composition, hours of care needs, wage effects of caregiving, endogenous threat points,
male partner’s parents, and altruistic preferences. Select numerical examples from model extensions are provided in Table 4, which is referenced throughout this section.

Table 4: Extensions: equilibrium change with parental care needs (France)

<table>
<thead>
<tr>
<th>% Change</th>
<th>Care</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_f$ $e_m$ $h_f$</td>
<td>$h_m$ $c_f$ $c_m$ $z_f$ $z_{gap}$ $\lambda_f$ $\lambda_m$ $\lambda_f/\lambda_m$</td>
<td></td>
</tr>
<tr>
<td>One earner</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>household</td>
<td></td>
<td>16.7</td>
</tr>
<tr>
<td>Single</td>
<td>-</td>
<td>---</td>
</tr>
<tr>
<td>female</td>
<td></td>
<td>11.6</td>
</tr>
<tr>
<td>household</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 hours of need</td>
<td>-</td>
<td>5.0</td>
</tr>
<tr>
<td>Wage</td>
<td></td>
<td>15.3</td>
</tr>
<tr>
<td>penalty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Threat</td>
<td></td>
<td>12.4</td>
</tr>
<tr>
<td>point model</td>
<td></td>
<td>12.4</td>
</tr>
<tr>
<td>In-law</td>
<td></td>
<td>37.3</td>
</tr>
<tr>
<td>caregiving</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Equilibrium response to care needs of 20 hours per week (10 hours per week in "10 hours of need" extension). Fixed $\theta$ in all extensions. Fixed labor supply in “Wage penalty” extension.

4.1 HOUSEHOLD COMPOSITION

While our main analyses focused on dual-earning partners, here we consider alternate household compositions. We begin with a household in which only one partner works in the formal labor market. How does a single-earner partnership respond to the onset of care needs? As the female labor supply margin is implicitly fixed in this case, couples respond in a similar fashion as the fixed labor supply model outlined above. Namely, if care needs are large enough, couples may choose to split time spent providing unpaid care. However, if the non-negativity constraint on unpaid care binds, and caregiving falls to a single partner, condition (13) again determines the allocation of home production across partners.

We again provide a numerical example for France to highlight the predicted welfare implications of unpaid caregiving for single-earner households. In this exercise, we recalibrate three parameters $\{v, \eta, \kappa\}$ to match average male hours worked and home
production hours of males and females in single-earner households in France. As female labor supply is zero and exogenous, we keep bargaining power fixed at the baseline value above ($\theta = 0.42$). We also keep the disutility weight on unmet care $\delta$ at the previously calibrated value to facilitate welfare cost comparisons across numerical examples.

The first row of Table 4 provides results for France for a single-earner household. In this example, the female provides 11.3 hours of unpaid care while the male does not provide any care. As market work is no longer substitutable across partners, home production becomes the primary margin of adjustment. In particular, females lower time spent in home production 16.8% while men increase home production 0.5%. The total welfare cost to the female is 36% of baseline market consumption while that of the male is 6%. This implies a welfare ratio of 15%. All of these welfare measures fall in-between the duel-earner baseline case with flexible versus fixed labor supply.

In addition to a single-earner household, we also consider a household comprised of a single working female. While a single female possesses full allocative power over her time and resources, she does not have the benefits of a bargained cooperative equilibrium enjoyed in the partnered household. Moreover, the single female shoulders the entire burden of parental care needs. We again recalibrate the relevant parameters $\{\nu, \kappa\}$ to match average hours spent in market and home production of single females in France. The second row of Table 4 provides the equilibrium response to 20 hours of care needs by the single female. Consumption, market work, and home production all fall by 11.6% and unmet care needs equal 5.2 hours. The welfare cost is 41% of baseline market consumption, which is significantly more than the 26% cost to the partnered female under the assumption of fixed bargaining power. However, the welfare burden of the single female is not substantially different from the partnered female under the assumptions of endogenous bargaining power or fixed labor supply. This highlights the potential for power dynamics or labor market rigidities to limit the insurance benefit of a household cooperative partnership.

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12 Data moments are 41.8 male hours worked, 12.0 hours of male home production, and 31.1 hours of female home production. Calibrated parameter values for $\{\nu, \eta, \kappa\}$ are $\{10.37, 0.06, 0.59\}$. Implicit in these calibrated values is that the observed behavior by single-earner households is rationalized in the model primarily by assigning higher utility weights on domestic goods and leisure relative to duel-earning couples.

13 Parameters are pinned down by the modified conditions $\kappa = h_f / e_f$ and $\nu = 1 / e_f(e_f + h_f)$. Data moments are 41.4 hours worked and 18.3 hours of home production. Calibrated parameter values for $\{\nu, \kappa\}$ are $\{5.52, 0.44\}$. 
4.2 HOURS OF CARE NEEDS

Our numerical exercises have assumed 20 hours of care needs generally resulting in endogenous care provision in the range of 10-15 hours a week. In practice, a majority of caregivers provide fewer hours of care. To explore the implications of lower care provision in the model, the third row in Table 4 provides summary results of the impact of 10 hours of care needs in the exogenous bargaining model for France. Provision of care falls to 8.1 hours with the remaining 1.9 hours of care needs unmet. Patterns are similar to the baseline example with more care hours but are smaller in magnitude. Namely, there is a significant reduction in female labor supply partially counteracted by a smaller increase in male hours works. The welfare cost to the female is roughly halved to 13% of baseline market consumption while that of the male falls to 8%. The welfare ratio increases to 63% implying, if anything, the welfare burden is slightly shifted towards the male partner with fewer hours of care needs.

4.3 WAGE PENALTY

Another assumption we have made is fixed wages for caregivers. However, caregiving may interfere with work even if there is no reduction in hours. For example, poorer physical health or mental strain could lower productivity at work and result in a wage penalty for caregivers. Empirical evidence on the size of such a wage penalty has been decidedly mixed. Using SHARE data, Bolin et al. (2008) find that caregiving does not generally reduce wages in Europe. However, other work has found wage penalties on the order of 0-10% (Bauer and Sousa-Poza, 2015). In this spirit, the fourth row in Table 4 shows the equilibrium response when labor supply is held fixed, but we introduce a 5% market wage penalty on caregivers in France. The overall welfare costs of the wage penalty are small relative to costs already captured in the baseline exercise. Specifically, the welfare cost to the female increases from 39% in the baseline to 40% with the introduction of the wage penalty. The welfare cost to the male increases a similar magnitude (from 4% to 6%) increasing the welfare ratio to 14%. The additional welfare costs are driven by a 2% reduction in household income and consumption as a result of the fall in caregiver wages.

4.4 THREAT POINT MODE

On the bargaining side, we adopted the collective approach of Chiappori (1992) allowing for a simple reduced form sharing rule. As an alternative, we could develop a more well-
defined cooperative Nash bargaining model of family partnerships where “threat points” are potentially endogenous to household decisions (e.g. McElroy and Horney, 1981). This approach is less general than the collective model but allows for better identification of endogenous bargaining power changes by modeling changes to a partner’s outside option. Moreover, it may be that threat points, and hence bargaining power, is influenced by potential earnings and not actual earnings (Pollak, 2005) as we have assumed in our benchmark endogenous bargaining power model. Here we consider such a model where parental care needs directly alter threat points. In our example, if a daughter must assume responsibility of taking care of her elderly parent, it lowers the value of her outside option (e.g. divorce) and hence lowers her equilibrium bargaining power. This could plausibly shift the welfare burden of caregiving towards the female similar to endogenous bargaining power under the collective model.

Consider a household that maximizes the welfare function

\[ (u(c_m, d_m, l_m) - u_m^o)^{1 - \theta} (u(c_f, d_f, l_f) - u_f^o)^{\theta} \]

where \( u_i^o \) is the “threat point” or utility level individual \( i \) could achieve if cooperation were abandoned (i.e. their outside option). The solution to this problem is an efficient Nash-bargained equilibrium (Gersbach and Haller, 2009). Bargaining power is determined jointly by the parameter \( \theta \) and the value of each partner’s threat point. We assume if cooperation breaks down the household dissolves and each member forms their own single member household (e.g. divorce). We continue to assume that care needs fall on daughters and therefore lowers the outside option value of the female. The males outside option is assumed to remain unchanged when care needs are introduced.

Unlike the collective model, the household will no longer follow a sharing rule in which consumption is divided based solely on relative bargaining weight \( \theta \). Instead, relative consumption is determined by the condition

\[
\frac{c_m}{c_f} = \frac{(1 - \theta)[u(c_f, d_f, l_f) - u_m^o]}{\theta[u(c_m, d_m, l_m) - u_f^o]},
\]

indicating that consumption share depends on both bargaining parameter \( \theta \) and each partner’s outside option.

As a numerical comparison, we recalibrate the weight \( \theta \) in the threat point model to again match average female hours worked for France of 37.1 hours. Equations (15)-(17) continue to hold for the threat point model so other parameters are held fixed at the

---

14 However, it could also be that lower labor supply decreases potential earnings through skill depreciation (or slower accumulation). For example, caregivers have reported an inability to reenter the job market after an extended period of time due to outdated knowledge (Carmichael et al., 2008).

15 Alternately, one could assume the household stays together but moves to a non-cooperative equilibrium; e.g. the separate spheres model (Lundberg and Pollak, 1993).

16 The calibrated \( \theta = 0.20 \). Recall that bargaining power in the threat point model is determined by this parameter as well as the value of the outside options.
baseline values ensuring the same set of data moments are matched. The last row of Table 4 provides summary results of the impact of 20 hours of care needs predicted by the calibrated threat point model. Overall, the pattern of results is similar to those predicted by the collective model with endogenous bargaining power. Disproportionate declines in female labor supply and consumption accompanied by about nine hours of unmet care needs. The welfare results are also nearly identical to the collective approach with a welfare cost to the male of just 1% of baseline market consumption while that of the female is 41%.

4.5. IN-LAW CAREGIVING

In our benchmark model we assumed the parent of the female partner was the source of care needs. This was to reflect that daughters are more likely to provide care than sons. Here we explore how results differ if we assume it is the male’s parent in need of care. The only change in the household problem is the weight placed on the disutility of unmet care, which now reflects the bargaining power of the male. However, Proposition 3 continues to hold, and the female continues to provide all unpaid care. The difference is now she is caring for her partner’s parent (e.g. her mother-in-law) instead of her own.\(^{17}\) The modified interior first-order condition for female unpaid care is given by:

\[
\theta \phi'(1 - e_f - h_f - z_f) = (1 - \theta) \omega'(z - z_f).
\]

Increases in female bargaining power now result in lower levels of care unless female work and home production drop dramatically or unmet care needs bind at zero.

The last row of Table 4 provides summary results of the impact of 20 hours of care needs for the male’s parent in the baseline model for France. Overall, care of the male’s parent results in more substantial reallocation of household time than provision of care to the female’s parent. As a result, 20 hours of care is provided and there are no unmet care needs. The welfare ratio reaches 95% as the male partner is able to use his high bargaining power to evenly distribute the burden associated with the care needs of his parent.

\(^{17}\) There is empirical support for females assuming the primary caregiver role for their in-laws (Allen et al., 2000). However, this empirical finding is not firmly established across all context (e.g. Chesley and Poppie, 2009). Note also the “female” and “male” labels in our model effectively reflect differences in average wages and bargaining power. In some households, in may be that the female has higher earnings and/or more bargaining power than the male. This may explain why in the data, we see some male’s providing care to their parents or in-laws. However, they provide much less overall than females.
4.6. CARING PREFERENCES

Our collective model assumes households form a cooperative equilibrium to the benefit of both partners. However, we have also assumed partners are egotistical in the sense that their welfare only depends on their own outcomes. An alternative approach, often termed “caring preferences”, assumes partners care about the level of utility the other achieves (Himmelweit et al., 2013). In this case, our household welfare function would be written:

\[(1 - \theta)[u(c_m, d_m, l_m) + \zeta u(c_f, d_f, l_f)] + \theta[u(c_f, d_f, l_f) + \zeta u(c_m, d_m, l_m)]\]

where \(\zeta \geq 0\) represents the weight each partner puts on the other’s utility. With \(\zeta = 0\) we return to our baseline egotistic preferences; while \(\zeta = 1\) yields the standard unitary model of the household.

The household objective function with caring preferences can equivalently be written:

\[(1 - \hat{\theta})u(c_m, d_m, l_m) + \hat{\theta}u(c_f, d_f, l_f)\]

where \(\hat{\theta} = \frac{1}{1+\zeta} [\theta + \zeta - \theta \zeta]\). As bargaining power was calibrated in our baseline numerical exercise to match hours worked, recalibrating the caring preferences version of the model would yield the same results only with \(\hat{\theta}\) equal to our baseline \(\theta\) values. In other words, the allocations of time and consumption across the household would be identical to our baseline results, only the interpretation of the bargaining weights would change. The weights \(\hat{\theta}\) represent a combination of bargaining power \(\theta\) and partner altruism \(\zeta\).

While the allocation of time and resources would remain the same as the baseline, caring preferences have implications for welfare results.\textsuperscript{18} In general, caring preferences will more evenly distribute the welfare burden. In France, for example, the exogenous bargaining power welfare ratio shifts from 57% with egotistic preferences to 84% with caring preferences and a caring weight \(\zeta = 0.5\). Similarly, the welfare ratio shifts from 9% to 62% with fixed labor supply. Clearly the magnitude of the welfare inequality of caregiving is sensitive to specification of caring preferences. This is not surprising given that by definition, the welfare ratio approaches one as the caring weight approaches one. However, the finding that labor market rigidities and/or endogenous power dynamics exacerbate the unequal distribution of welfare remains.

\textsuperscript{18} In particular, welfare for the male and female partner would now be given by \(\lambda_m = \exp(u(c_{m}, d_{m}, l_{m}) + \zeta u(c_{f}, d_{f}, l_{f}) - \zeta \omega(z_{m} - z_{m} - z_{f}) - u(c_{m}, d_{m}, l_{m}) - \zeta u(c_{f}, d_{f}, l_{f})) - 1\) and \(\lambda_f = \exp(u(c_{f}, d_{f}, l_{f}) - \omega(z_{m} - z_{f}) + \zeta u(c_{m}, d_{m}, l_{m}) - u(c_{f}, d_{f}, l_{f}) - \zeta u(c_{m}, d_{m}, l_{m})) - 1\).
5. CONCLUSIONS

With the aid of a simple collective model of intra-household bargaining we analyzed the time and consumption allocation decisions and welfare costs associated with unpaid parental caregiving. Our results suggest that with more equal bargaining power and fully flexible labor markets, the welfare burden of caregiving can be more evenly distributed across household partners and unmet care needs of the elderly parent can be minimized. However, with endogenous bargaining power, the welfare burden can be greatly skewed towards the primary caregiver and significant unmet care needs can arise. This is due to a “triple burden” of market work, home production, and caregiving. Under this scenario, government policies subsidizing long-term care could decrease the welfare gap within a household by providing financial relief and improving the bargaining position of the caregiver.

We also demonstrated that labor market rigidities can exacerbate the total welfare cost of unpaid caregiving to the household as well as the unequal distribution of the burden. This implies policies that promote flexibility in number of working hours, such as caregiver leave or part-time options, could provide substantial relief to caregivers and care recipients. Note that we have limited our analysis to labor market rigidities that restrict the ability of partners to adjust their number of hours worked. Other types of rigidities could also have welfare implication by limiting the ability to freely allocate time across the day or week. For example, a fixed 9 to 5 work schedule may limit the ability to provide care at certain times of day. In this case, policies that promote the ability to adjust work schedules, such as flexible shifts or flextime, could improve outcomes for care recipients and caregivers even if they choose to keep total hours worked unchanged.

While our simple static model demonstrates the potential quantitative influence of bargaining power within a household, other considerations are warranted if robust counterfactual policy experiments are desired. Empirical evidence suggests unpaid caregiving lowers labor supply on both the extensive and intensive margins. Incorporating partially indivisible labor supply and additional heterogeneities across households could yield additional insights. Dynamic considerations could also play an important role. For example, the expected persistence in unpaid caregiving could have implications in the presence of labor market frictions or re-employment costs. It is also important to highlight that total welfare costs may be underestimated in our model if caregiving is accompanied by additional market expenses (e.g. food, medicine, etc.). Negative effects on caregiver health have also been well documented (Bauer and Sousa-Poza, 2015), suggesting the welfare burden may be further skewed towards those actually providing care. While most existing empirical research has focused on labor market outcomes, our results also suggest that future work should examine the empirical link between caregiving and total time allocation patterns, including other forms of home production. Finally, while unpaid care continues to play a vital role in most countries, increased reliance on formal care markets is an important additional margin for consideration in future work.
REFERENCES


### 7. APPENDIX: PROOFS OF PROPOSITIONS

#### 7.1 PROPOSITION 1

Denote the Lagrange multiplier on the household budget constraint $\lambda_c$ and on the home production constraint $\lambda_d$. Then the first-order conditions for female and male market consumption are given by $\frac{(1-\theta)}{c_m} = \lambda_c$ and $\frac{\theta}{c_r} = \lambda_c$. These conditions combined with (2) yields (4). The first order conditions for domestic consumption are given by $\frac{(1-\theta)\kappa}{d_m} = \lambda_d$ and $\frac{\theta\kappa}{d_r} = \lambda_d$. These conditions combined with (3) yields (5).

#### 7.2 PROPOSITION 2

First-order conditions for $e_f, e_m, h_f, h_m$ are given by:

\[
\phi'(1-e_r h_f) = \frac{\gamma}{\theta(e_m + ye_f)} \tag{19}
\]

\[
\phi'(1-e_m h_m) = \frac{1}{(1-\theta)(e_m + ye_f)} \tag{20}
\]

\[
\phi'(1-e_r h_f) = \frac{\kappa(1-\alpha)h_f^{\gamma-1}}{\theta(ah_m^n + (1-\alpha)h_f^n)} \tag{21}
\]

\[
\phi'(1-e_m h_m) = \frac{\kappa h_m^{\gamma-1}}{(1-\theta)(ah_m^n + (1-\alpha)h_f^n)} \tag{22}
\]

where we have used the implied sharing rule conditions: $c_r = \theta(e_m + ye_f)$ and $d_r = \theta(ah_m^n + (1-\alpha)h_f^n)^{\frac{1}{n}}$. Solving for $e_m(e_f, h_f)$ and $h_m(e_f, h_f)$, the system reduces to:

\[
\phi'(1-\varphi_2 h_f + ye_f) = \frac{\kappa(1-\alpha)}{\gamma(1-\theta)\varphi_2 h_f} \tag{23}
\]

\[
\phi'(1-e_r h_f) = \frac{\kappa(1-\alpha)}{\theta \varphi_2 h_f} \tag{24}
\]

where

\[
\varphi_1 = \frac{\gamma\alpha}{\kappa(1-\alpha)} \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{\eta}} + \frac{\gamma}{\kappa} + \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{\eta}} > 0 \tag{25}
\]
\[ \varphi_2 = a \left[ \frac{\gamma \alpha}{1-\alpha} \right]^{1/\gamma} + (1-\alpha) > 0. \]  

Using these together with Cramer’s rule yields

\[ \frac{\partial e_f}{\partial \vartheta} = \frac{\left[ \frac{1}{\gamma(1-\theta)^2} + \Phi_m \left( \frac{1}{\gamma} \right) \right] \frac{\kappa (1-\alpha)}{\varphi_2 h_f^2}}{(1 + \frac{\varphi_2}{\gamma \kappa a} \left[ \frac{\gamma \alpha}{1-\alpha} \right]^{1/\gamma} \frac{1}{\gamma}) \Phi_m \left( \frac{1}{\gamma} \right) \Phi_m \left( \frac{1}{\gamma} \right) \left( \frac{1}{\gamma(1-\theta)^2} + \Phi_m \left( \frac{1}{\gamma} \right) \right) \frac{\kappa (1-\alpha)}{\varphi_2 h_f^2}} < 0. \]

Moving to male labor supply, using conditions (19)-(22) and solving for \( e_f(e_m, h_f) \) and \( h_m(e_m, h_f) \), the system reduces to:

\[ \begin{align*}
\Phi \left( 1 - \left( 1 + \frac{\varphi_2}{\gamma \kappa a} \right) h_f + \frac{1}{\gamma} e_m \right) &= \frac{\gamma \kappa a}{\varphi_2 h_f} \\
\Phi \left( 1 - e_m \left[ \frac{\gamma \alpha}{1-\alpha} \right]^{1/\gamma} h_f \right) &= \frac{\kappa a}{(1-\theta) \varphi_2 h_f}
\end{align*} \]

where \( \varphi_2 \) is as defined in (26). Using these together with Cramer’s rule yields

\[ \frac{\partial e_m}{\partial \vartheta} = \frac{\left[ \frac{1}{\gamma(1-\theta)^2} + \Phi_m \left( \frac{1}{\gamma} \right) \right] \frac{\gamma \alpha}{1-\alpha} \left[ \frac{\gamma \alpha}{1-\alpha} \right]^{1/\gamma} \frac{1}{\gamma} \Phi_m \left( \frac{1}{\gamma} \right) \Phi_m \left( \frac{1}{\gamma} \right) \left( \frac{1}{\gamma(1-\theta)^2} + \Phi_m \left( \frac{1}{\gamma} \right) \right) \frac{\kappa a}{\varphi_2 h_f^2} > 0. \]

### 7.3 Proposition 3

Assume \( y < 1 \) and \( e_f > 0 \). Denote the Lagrange multiplier on the non-negativity constraint on female unpaid care \( \lambda_f \), on male unpaid care \( \lambda_m \), and on the constraint \( z \geq z_f + z_m \) as \( \lambda_z \). Then the first-order conditions for female and male unpaid care are given by

\[ \theta \Phi \left( 1 - e_f h_f z_f \right) = \lambda_f \lambda_z - \theta w \left( z - z_m - z_f \right) \] and \( (1-\theta) \Phi \left( 1 - e_m h_m z_m \right) = \lambda_m \lambda_z - \theta w \left( z - z_m - z_f \right). \]

Combined this yields

\[ \theta \Phi \left( 1 - e_f h_f z_f \right) - \lambda_f = (1-\theta) \Phi \left( 1 - e_m h_m z_m \right) - \lambda_m. \]

Given an interior solution for \( e_f \), this condition may be rewritten using the first-order conditions for male and female labor supply as

\[ (1-\gamma) \lambda_c + \lambda_e = \lambda_m - \lambda_f > 0 \] \hspace{1cm} (29)
where $\lambda_{e_m} \geq 0$ is the multiplier on the non-negativity constraint on $e_m$ and we know that the household budget constraint holds with equality so $\lambda_c > 0$. Complementary slackness implies $\lambda_m, \lambda_f \geq 0$ and an interior solution requires that at most one non-negativity constraint binds: $\lambda_m = 0$ or $\lambda_f = 0$. Thus, it must be the case that $\lambda_m > 0$ and $\lambda_f = 0$ in order for (29) to hold. This shows that the female supplies all unpaid care. If $\gamma > 1$ and $e_m > 0$, equation (29) instead becomes

$$ (\gamma - 1)\lambda_c + \lambda_{e_f} = \lambda_f - \lambda_m > 0. $$

Analogous arguments show that the male supplies all unpaid care in this case.

7.4 PROPOSITION 4

With unpaid caregiving, the two-equation system (23)-(24) becomes

$$\begin{align*}
\phi'(1 - \varphi_1 h_f + \gamma e_f) &= \frac{\kappa(1 - \alpha)}{\gamma(1 - \theta)\varphi_2 h_f} \\
\phi'(1 - e_f - h_f z_f) &= \frac{\kappa(1 - \alpha)}{\theta\varphi_2 h_f}
\end{align*} \quad (30)$$

where $\varphi_1$ and $\varphi_2$ are as defined in (25) and (26). Using these together with Cramer’s rule yields

$$\frac{\partial e_f}{\partial z_f} = \frac{\kappa(1 - \alpha)}{(\gamma + \varphi_1)\varphi_2^2 h_f^2} \phi''(l_f) - \varphi_1 \phi''(l_m) \phi''(l_f) - \frac{\gamma \kappa \alpha}{\theta \varphi_2 h_f} < 0. $$

Moving to male labor supply, (27)-(28) become

$$\begin{align*}
\phi'(1 - (1 + \frac{\varphi_2}{\gamma \kappa \alpha}) h_f + \frac{1}{\gamma} e_m z_f) &= \frac{\gamma \kappa \alpha}{\theta \varphi_2 h_f} \\
\phi'(1 - e_m - \frac{\gamma \alpha}{1 - \alpha} \frac{1}{1 - \eta} h_f) &= \frac{\kappa \alpha}{(1 - \theta) \varphi_2 h_f}
\end{align*} \quad (31)$$

where $\varphi_2$ is as defined in (26). Using these together with Cramer’s rule yields

$$\frac{\partial e_m}{\partial z_f} = \frac{\phi''(l_m) \phi''(l_f) \left[ \frac{\gamma \alpha}{1 - \alpha} \frac{1}{1 - \eta} \cdot \frac{\kappa \alpha}{(1 - \theta) \varphi_2 h_f^2} \phi''(l_f) \right]}{\left(1 + \frac{\varphi_2}{\gamma \kappa \alpha} + \left[ \frac{\gamma \alpha}{1 - \alpha} \frac{1}{1 - \eta} \frac{1}{\gamma} \right] \phi''(l_m) \phi''(l_f) - \left( \phi''(l_f) \frac{1}{\gamma(1 - \theta)} + \phi''(l_m) \frac{\gamma}{\theta} \right) \frac{\kappa \alpha}{\varphi_2 h_f^2}} > 0. $$

Moving to female home production, (30)-(31) can again be used with Cramer’s rule to find

$$\frac{\partial h_f}{\partial z_f} = \frac{-\gamma \varphi''(l_m) \varphi''(l_f)}{(\gamma + \varphi_1) \varphi''(l_m) \varphi''(l_f) - \left( \varphi''(l_f) \frac{1}{\gamma(1 - \theta)} + \varphi''(l_m) \frac{\gamma}{\theta} \right) \frac{\kappa(1 - \alpha)}{\varphi_2 h_f^2}} < 0. $$
Finally, the combined optimally condition for home production remains identical to the case without unpaid caregiving: \( h_m = \left[ \frac{\nu}{1-\alpha} \right]^{\frac{1}{\gamma}} h_f \). Thus it is clear that \( \frac{\partial h_m}{\partial \alpha} > 0 \) and hence \( \frac{\partial h_m}{\partial z_f} < 0 \). This also shows that the proportional response to increased unpaid caregiving will be the same for male and female home production.